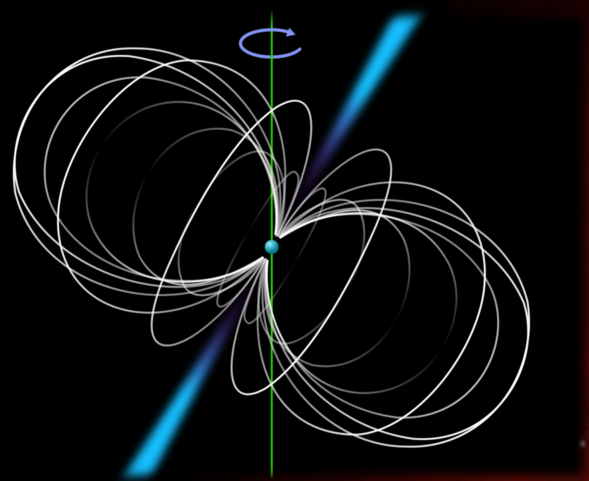
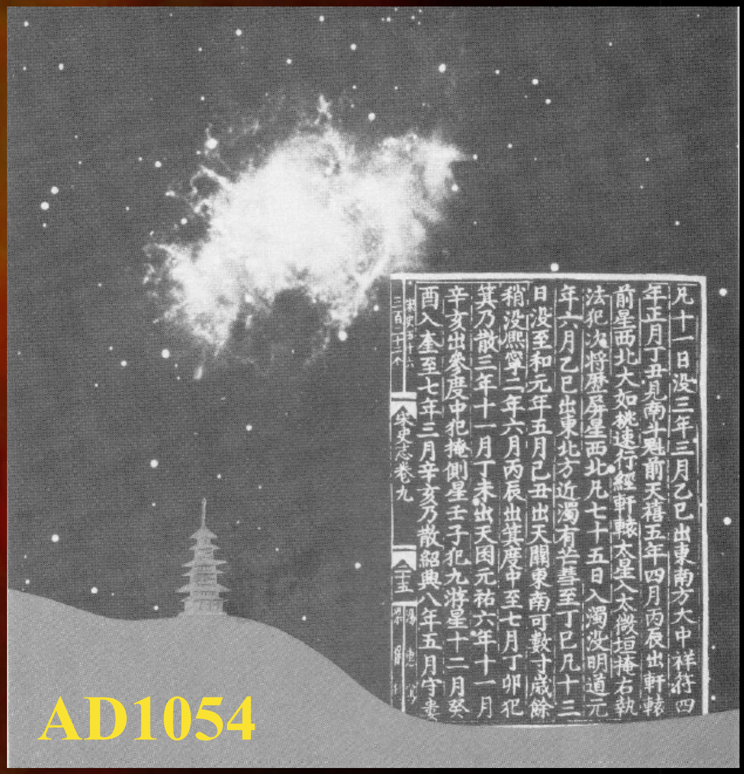
Introduction to Neutron Stars I: Fundamentals and Observations





Yong Gao

Jürgen Ehlers Spring School

AEI, Potsdam, March 3-14, 2025

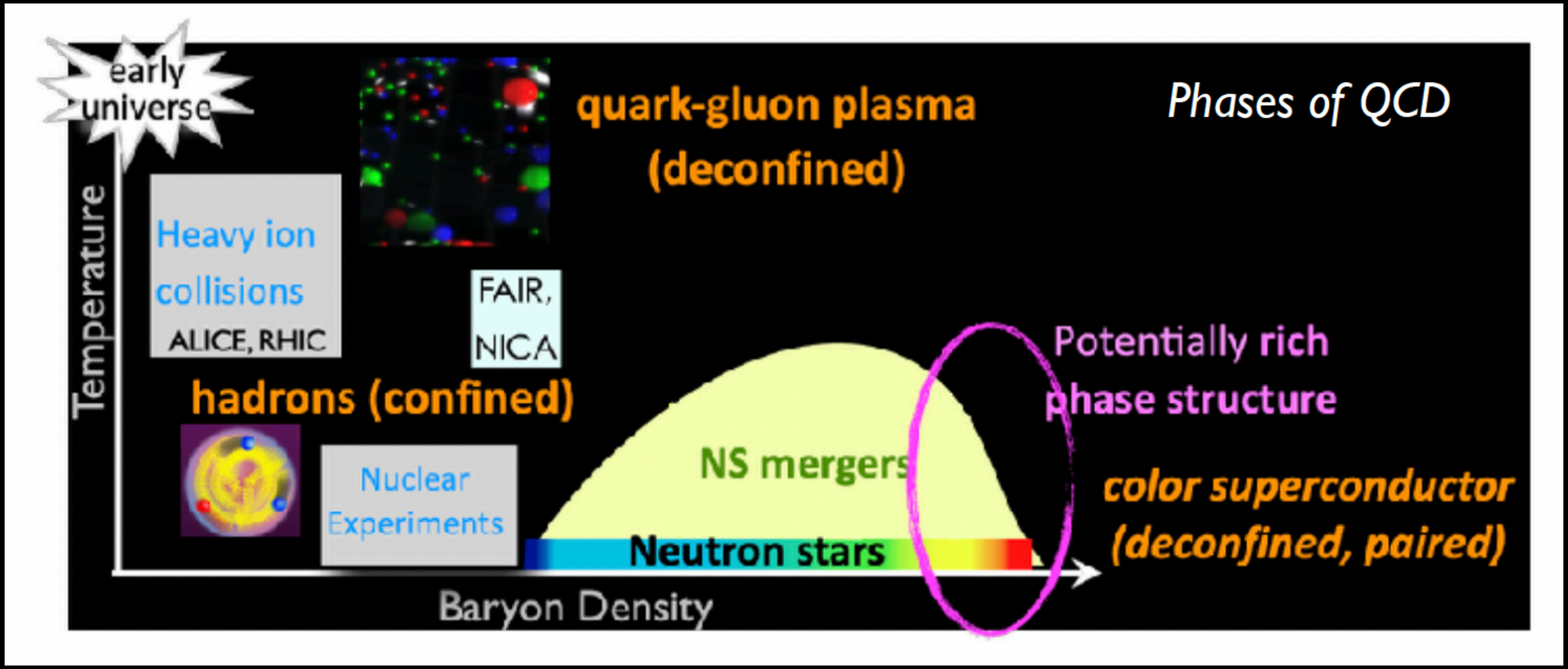
Neutron star as laboratories of extreme physics

- Formed from core collapse supernova, gigantic nucleus with $A = 10^{57}$ baryons

  $M \sim 1.4\, M_{\odot}$, $R \sim 10\,\mathrm{km}$, release energy $E_{\mathrm{g}} \approx GM^2/R \approx 10^{53}\mathrm{erg}$

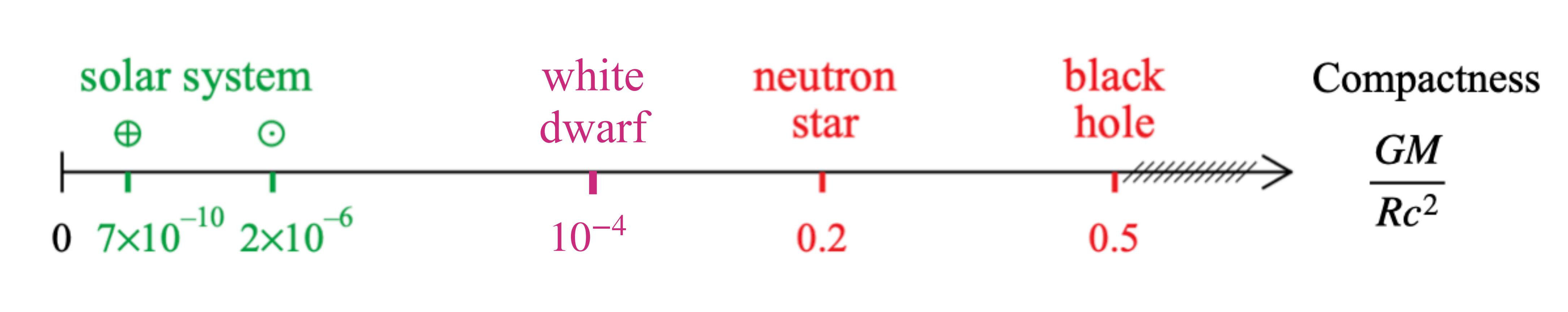
- Four fundamental forces come into play

  1. Strong interaction determines the structure: $\bar{\rho} \sim (2-3)\rho_0$, all humanity can be squashed down to a sugar-sized piece



Neutron star as laboratories of extreme physics

- Four fundamental forces come into play
  1. Strong interaction determines the composition and state
  2. Strong gravity, large compactness, 3 times of Schwarzschild radius



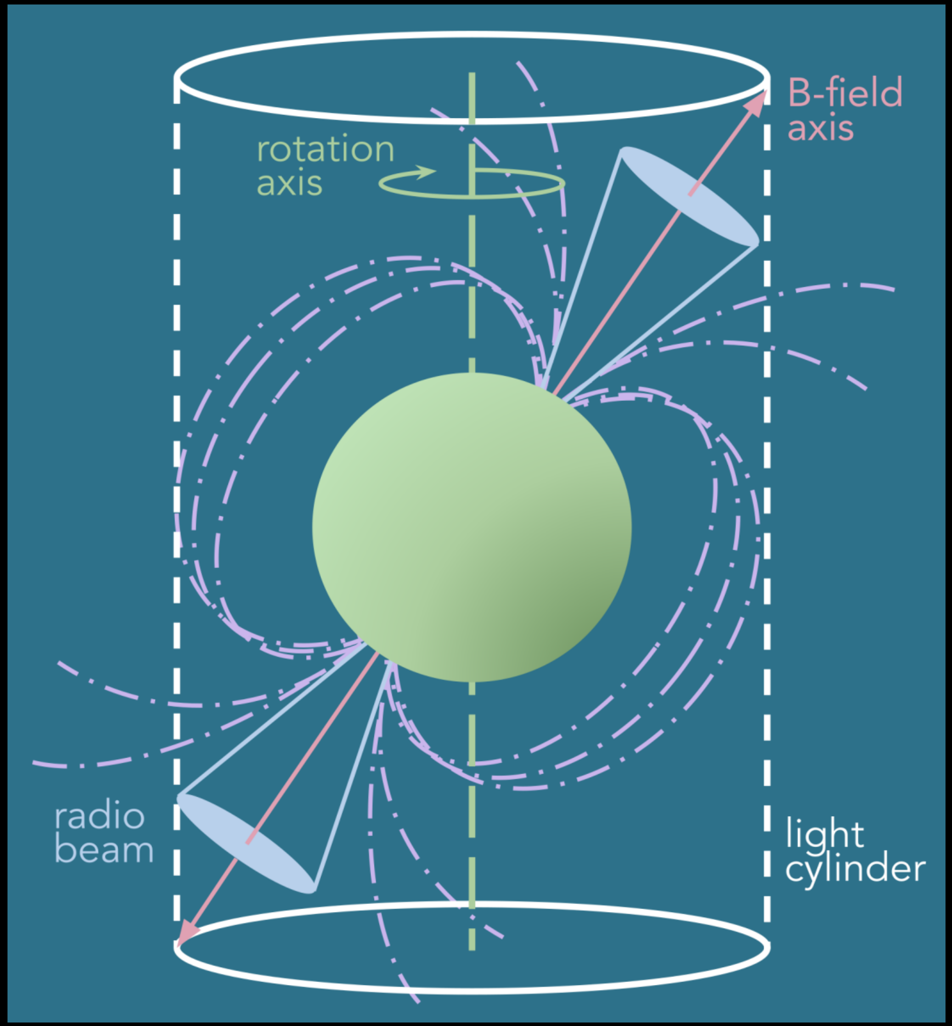
(1). Need General Relativity to describe the structure, e.g., TOV for spherical star

(2). Affect the motion of particles or celestial bodies within it and in its vicinity：the transport of neutrinos from the interior to the exterior, electromagnetic radiation from the surface and surrounding region, the motion of accreting matter, and the orbital dynamics of binary systems.

(3). Important source of gravitational waves: deformed or oscillating NSs, supernova explosions, and the inspiral and merger of binaries consists of NSs

Neutron star as laboratories of extreme physics

- Four fundamental forces come into play
  1. Strong interaction determine the composition and state
  2. Strong gravity, large compactness, 3 times of Schwarzschild radius
  3. Strong EM field: make pulse of pulsars, laboratory for QED processes



Schematic figure for pulsar

(1). Most pulsars have magnetic field on the order of $10^{12}\,\mathrm{G}$

(2). Unipolar induction: $E \sim 10^{10} - 10^{11}\,\mathrm{V/cm}$

(3). Exterior is dominant by electromagnetic forces

$$\frac{GMm}{R^2} / \frac{e\Omega RB}{c} \approx 10^{-9}$$

Neutron star as laboratories of extreme physics

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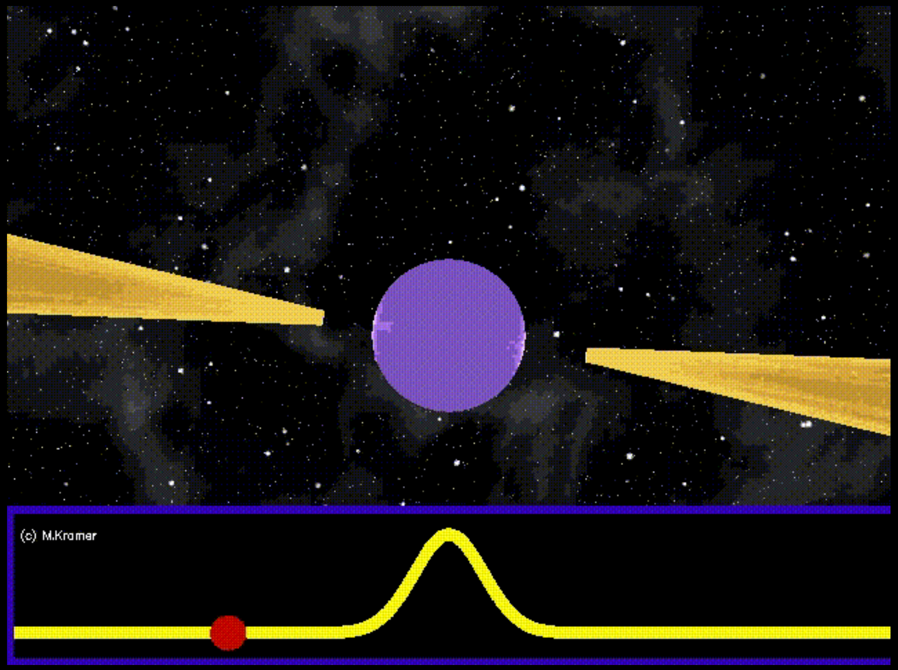
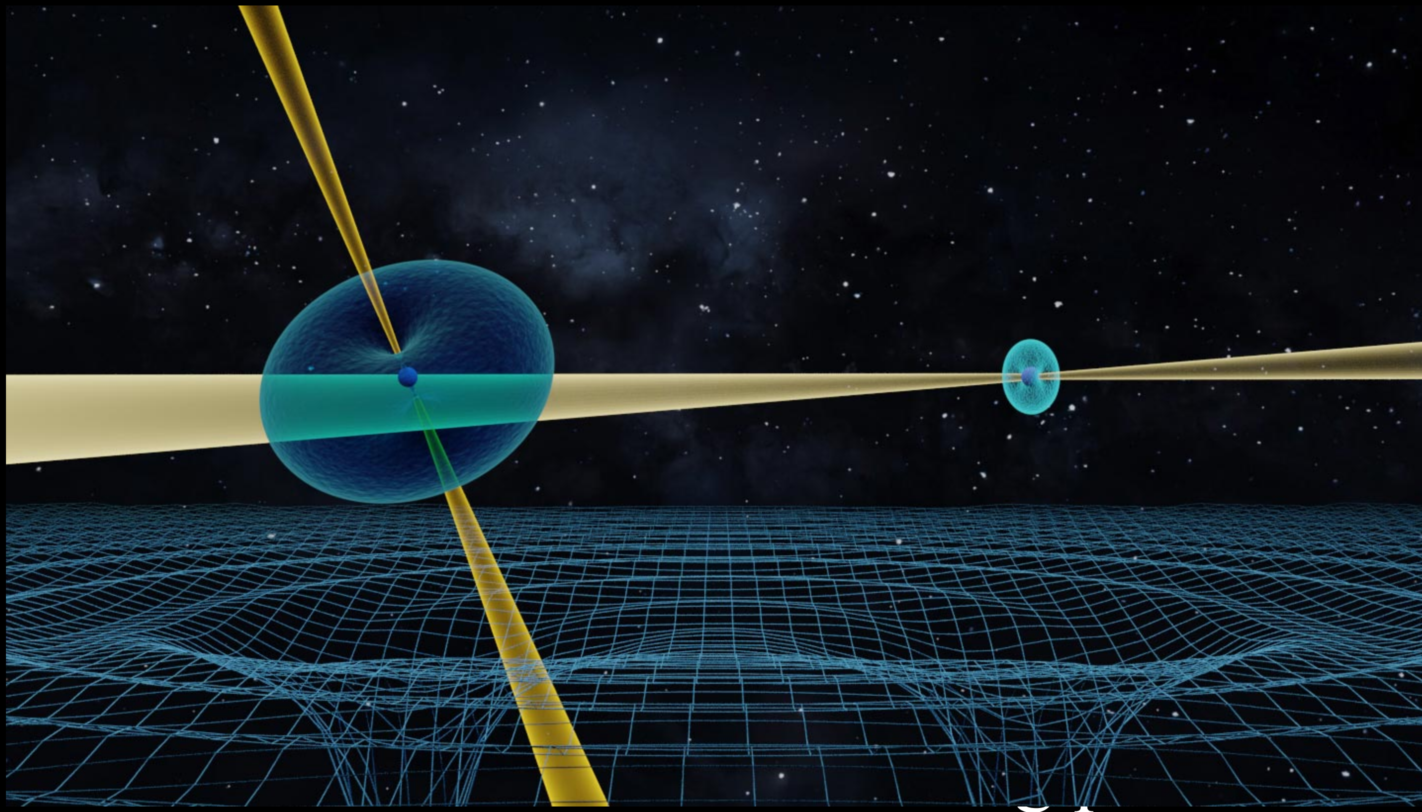
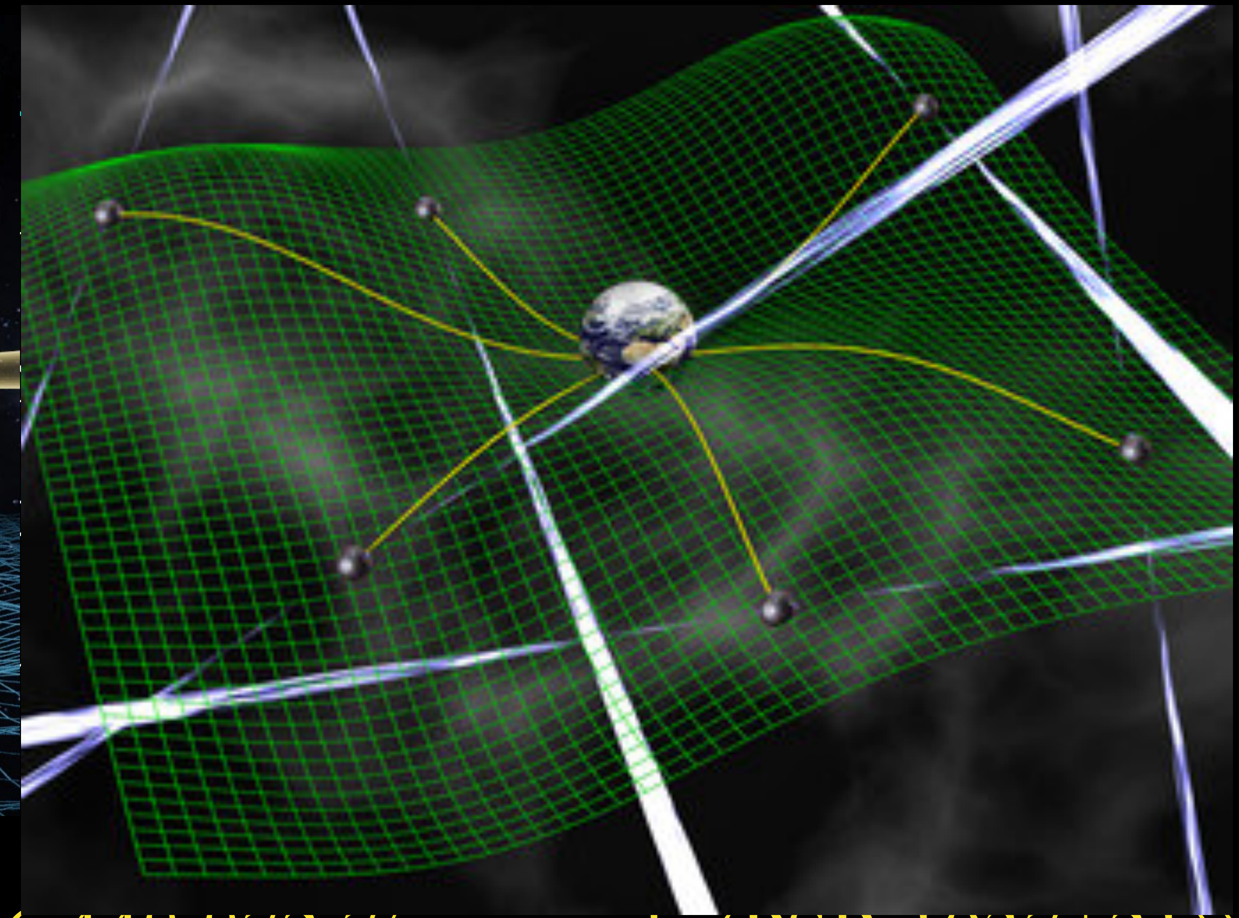
  1. Strong interaction: determine the composition and state
  2. Strong gravity, large compactness, 3 times of Schwarzschild radius
  3. Strong EM field: make pulse of pulsars, laboratory for QED processes
  4. Weak interaction: cooling processes

- Pulsars are clocks (e.g., P = 0.001 557 806 448 872 75 seconds (PSR 1937+21))



1. Study the spin evolution (e.g., glitch and spin-down mechanism)
2. If in binary system, accurately determine the orbital parameters and mass of NSs
3. Detect Cosmological GW backgrounds
4. Interstellar navigation





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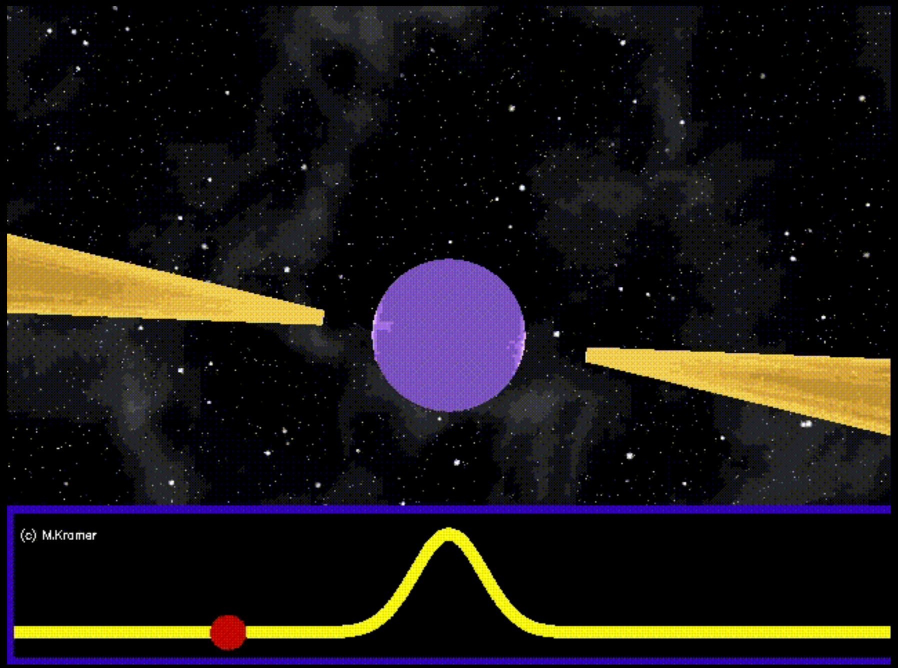

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Neutron star as laboratories of extreme physics

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  2. Strong gravity, large compactness, 3 times of Schwarzschild radius
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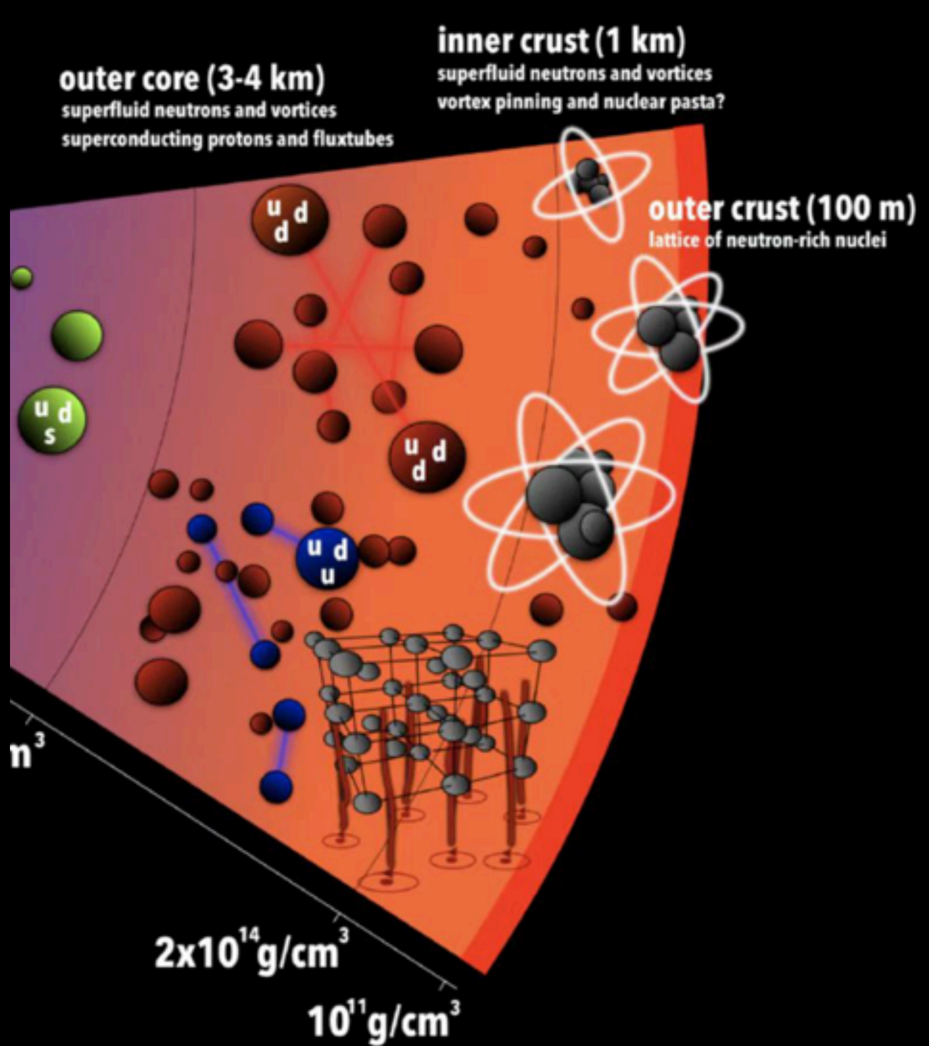
- Pulsars are high-precision clocks (pulsar timing)
- Low-temperature physics

Temperature much smaller than Fermi temperature

Neutron pairing —> superfluid

Pulsar glitch may be caused by the interaction of the superfluid and the solid crust



Neutron star as laboratorie

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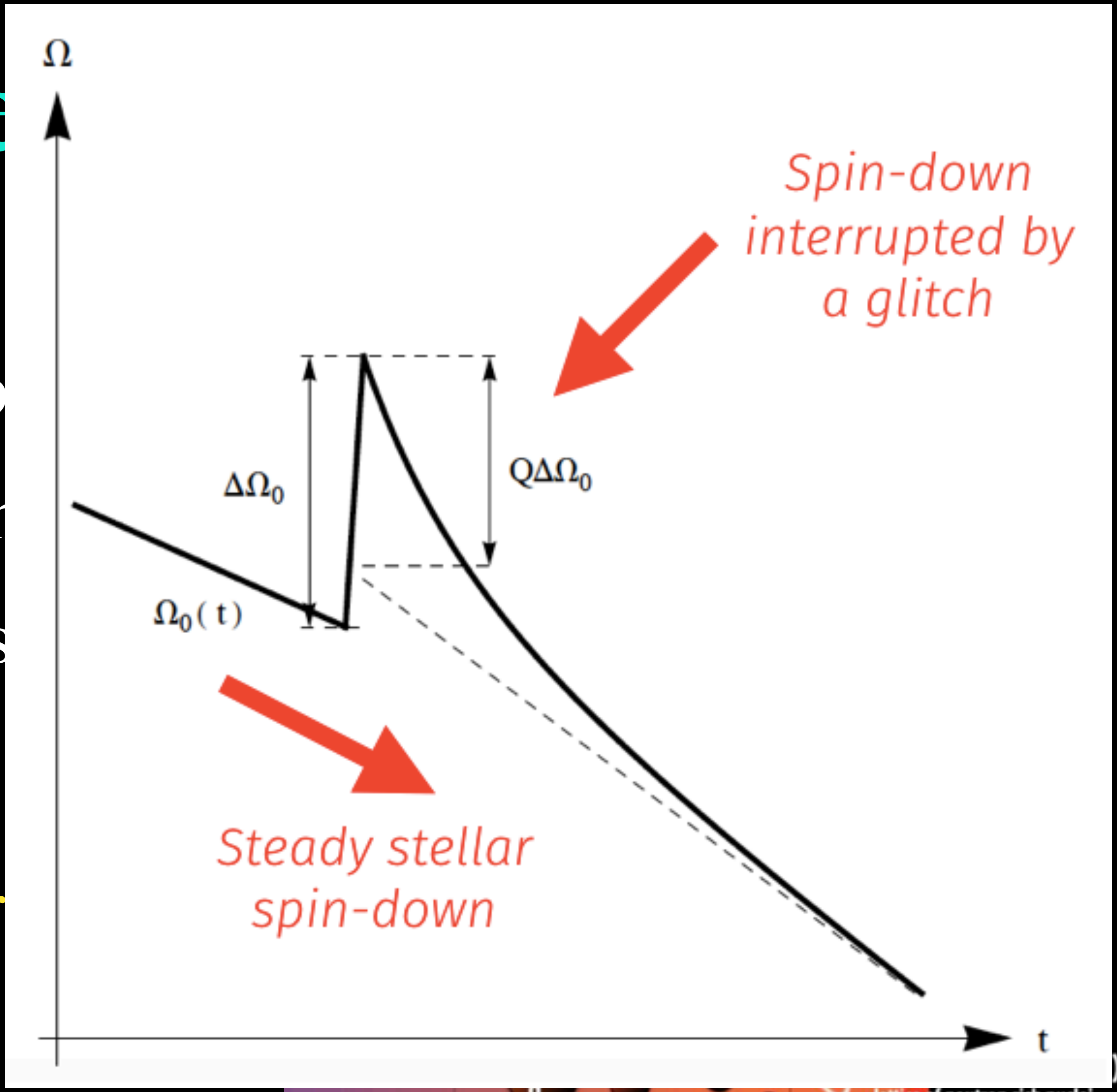
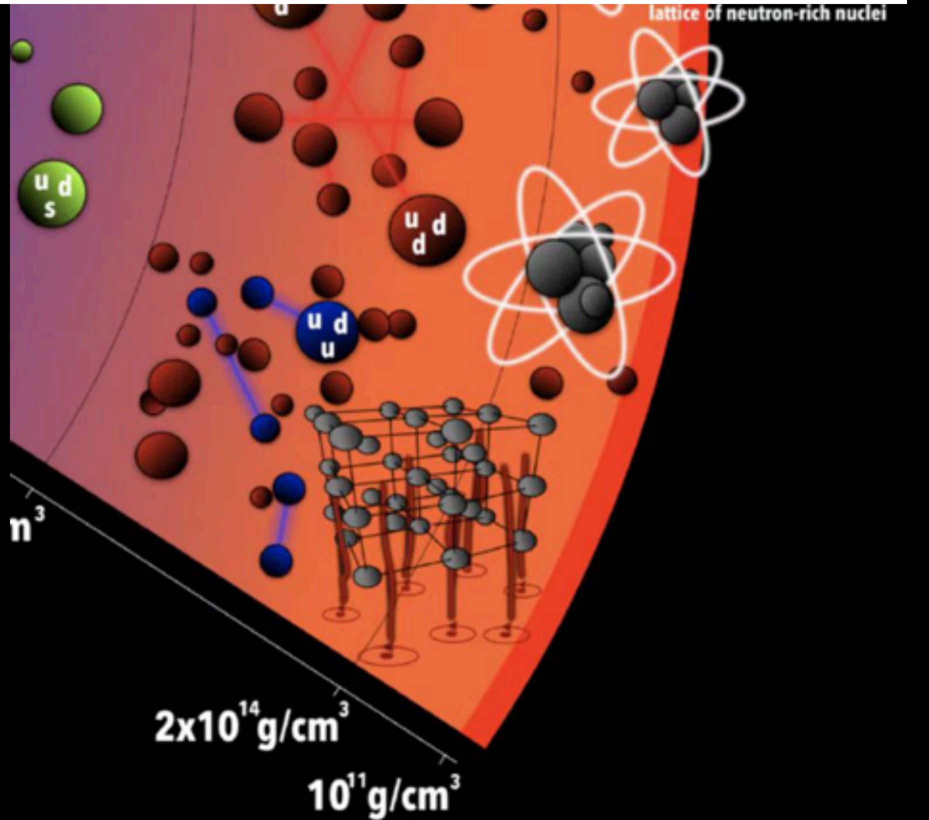




Pines theorem: Neutron stars are superstars

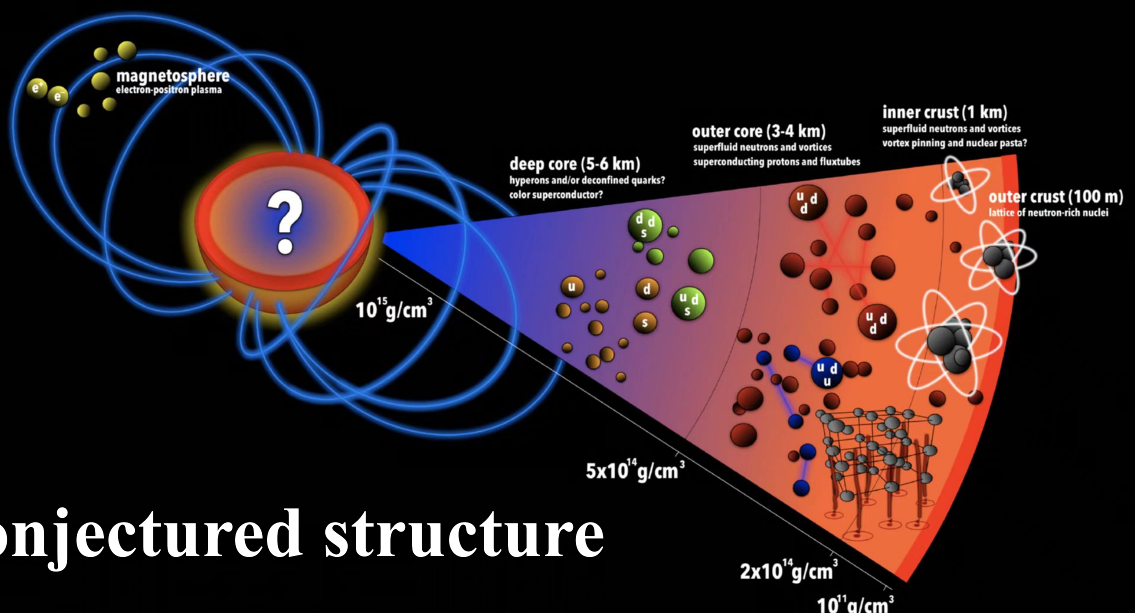
They are indeed superdense ($\bar{\rho} \gtrsim 2.8 \times 10^{14}$ g cm^{-3}), endowed with superstrong gravity ($GM/Rc^2 \sim 0.2 - 0.3$, in need of GR). They are superfast rotators ($\nu \sim 716$ Hz) and superprecise clocks (in need of ≥ 10 digits), but also superglitching objects ($\Delta\nu_{max} \sim 10^{-5}$ Hz). NSs possess superstrong magnetic fields ($B \sim 10^{12} - 10^{13}$ G). NS matter is partially superconducting and/or superfluid ($T \ll T_F \approx 10^{12} (\rho/\rho_0)^{2/3}$ K). In all, NSs are superrich in the physics involved (all four fundamental forces; Nuclear & Particle & Condensed Matter & Plasma & Magnetohydrodynamics & GR & Radio/Optical/X-ray/γ-ray & Neutrino & GW Physics etc.). ...and they are born in supernovae!

Laboratories for extreme physics!

In this lecture, I will talk about

- **A journey into the interior of NS**

Composition and states of matter, a.k.a. equation of state (EOS)



Conjectured structure

From $1\ \mathrm{g\,cm^{-3}}$ to $10^{14} - 10^{15}\ \mathrm{g\ cm^{-3}}$

Gas at the surface, nuclear matter at the centre

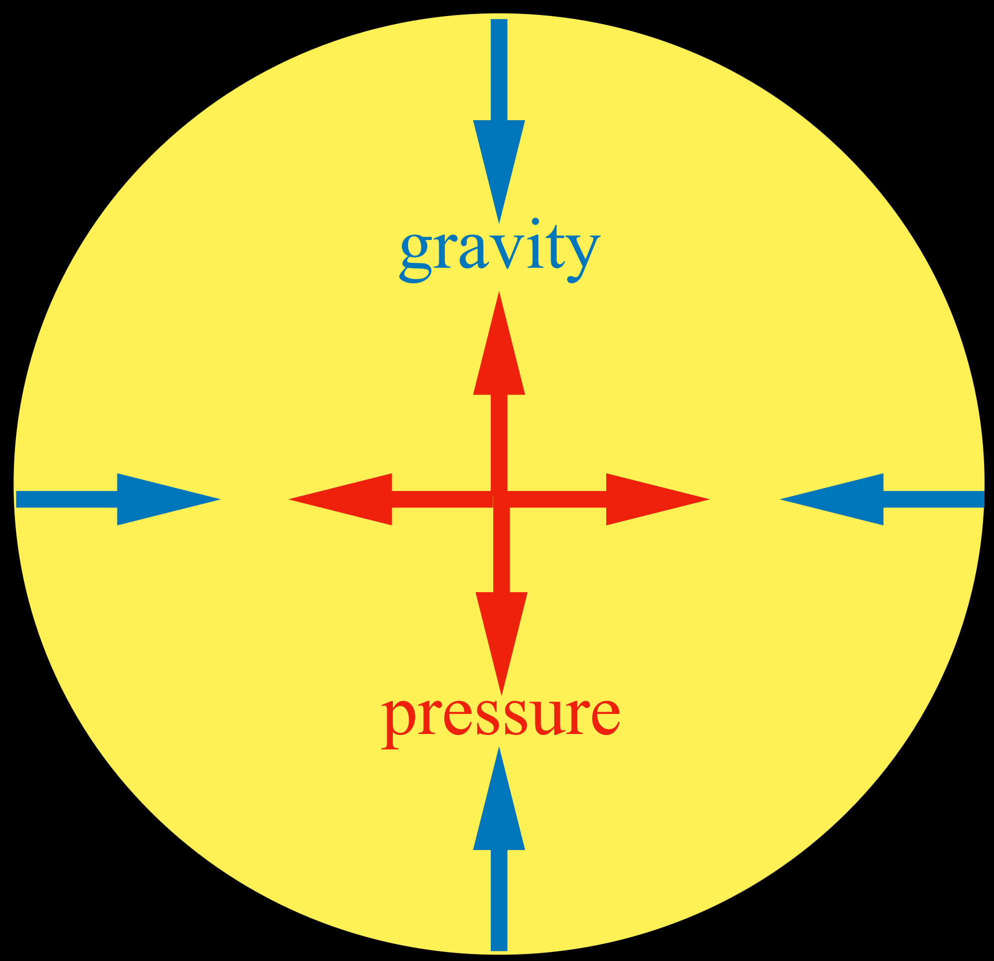
How this transformation occurs sets the internal structure of the star

- **Global properties: mass, radius…**

- **Observations: Many faces of NSs and constraints on the EOS**

Structures of astronomical bodies

balance between pressure and gravity



$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}$$

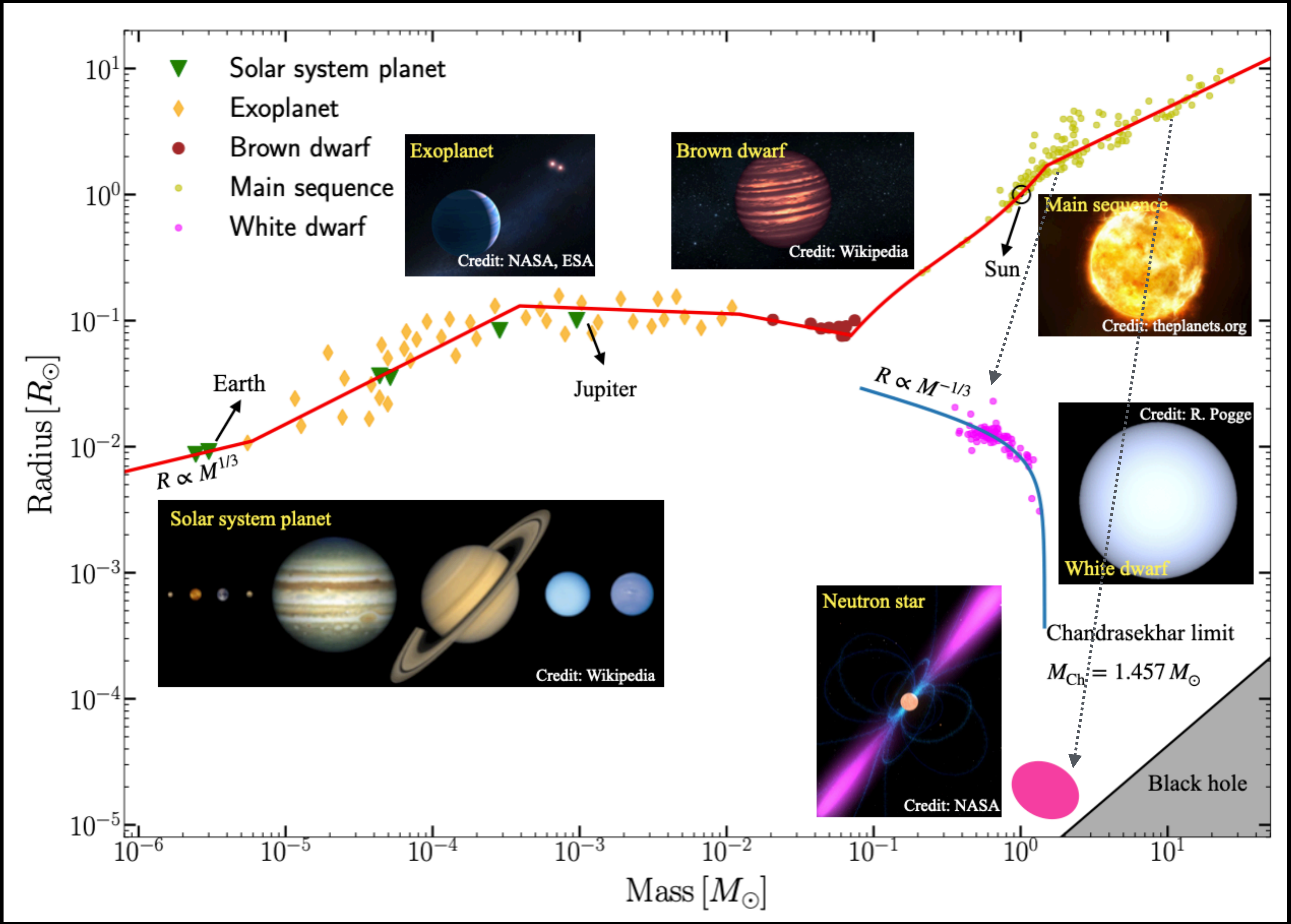
$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Equation of state: $p = p(\rho, \ldots)$

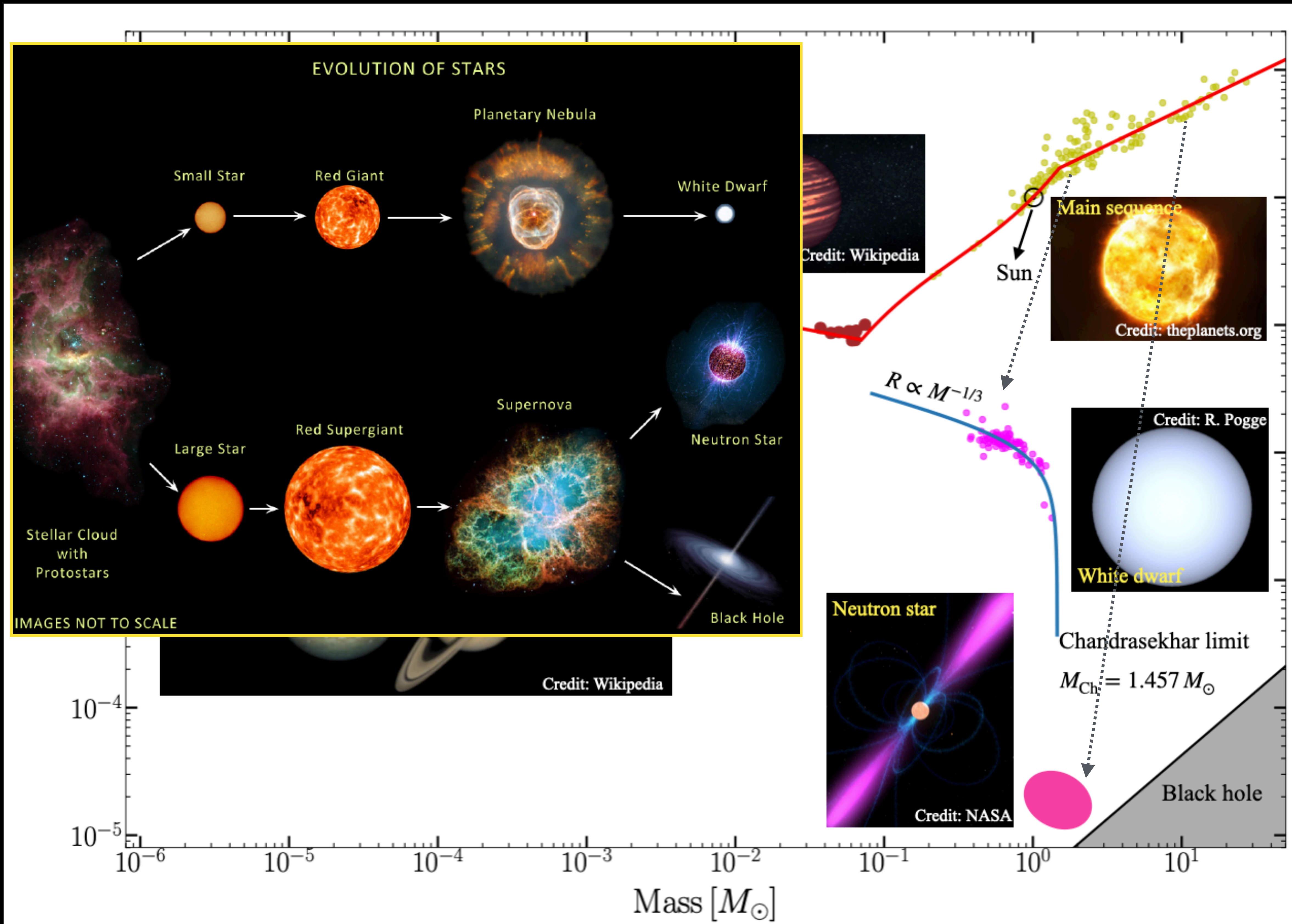
Gravity is universal and global, cannot be screened

Pressure is local, determined by the microphysics (composition and state of matter)

Mass-radius relation of astronomical bodies

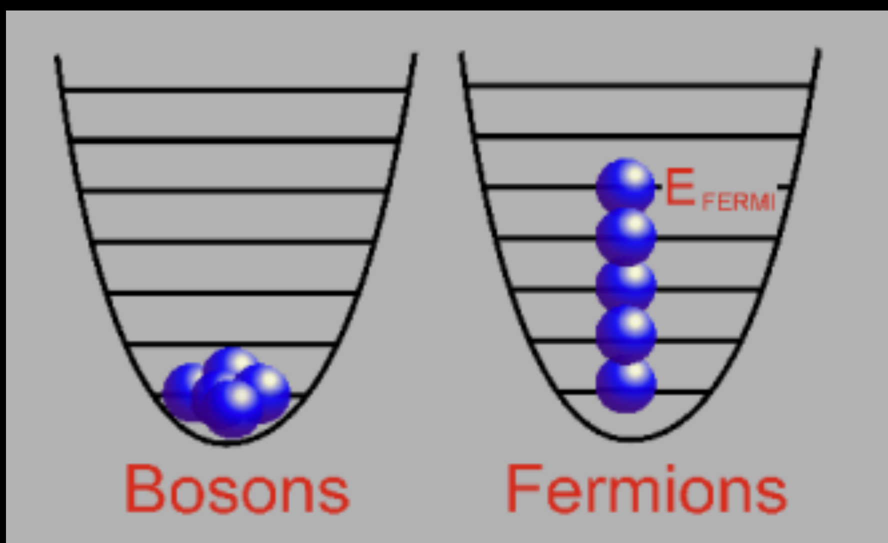


Mass-radius relation of astronomical bodies

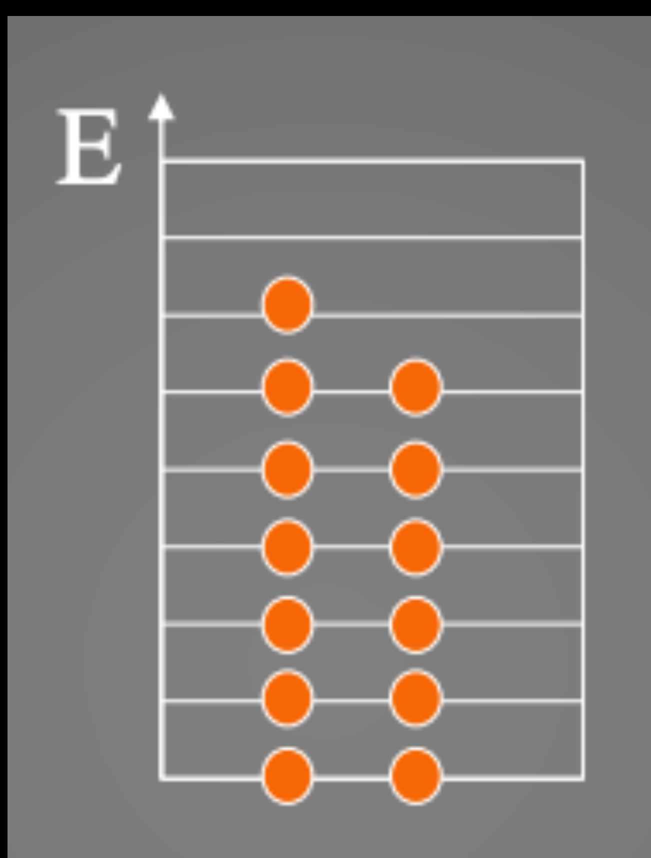


On dense matter

Let’s first take a look at white dwarf

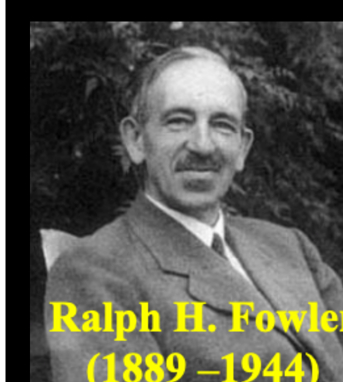


- Quantum mechanics is important for the description of a gas at low temperature and high density.
- Degenerate matter: matter which has sufficiently high density that the dominant contribution to its pressure (called degeneracy pressure) rises from the Pauli exclusion principle

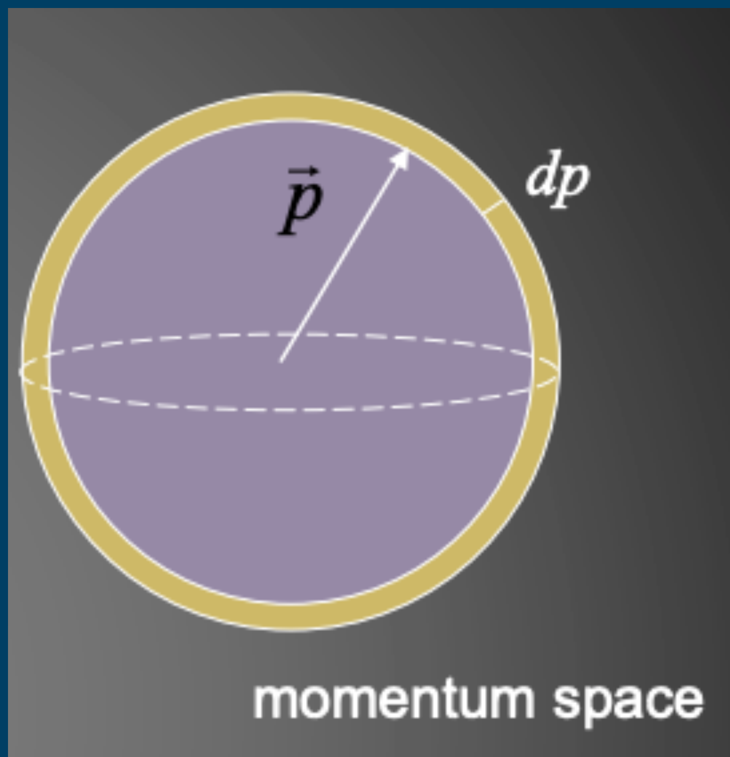


On Dense Matter. By R. H. Fowler, F.R.S. **1926**

§ 1. *Introductory*.—The accepted density of matter in stars such as the companion of Sirius is of the order of 10^5gm./c.c. This large density has already given rise to most interesting theoretical considerations, largely due to Eddington. We recognise now that matter can exist in such a dense state if it has sufficient *energy*, so that the electrons are not bound in their ordinary atomic orbits of atomic dimensions, but are in the main free—with sufficient energy to escape from any nucleus they may be near. The density of such “ energetic ” matter is then only limited *a priori* by the “ sizes ” of electrons and atomic nuclei. The “ volumes ” of these are perhaps 10^{-14} of the volume of the corresponding atoms, so that densities up to 10^{14} times that of terrestrial materials may not be impossible. Since the greatest stellar densities are of an altogether lower order of magnitude, the limitations imposed by the “ sizes ” of the nuclei and electrons can be ignored in discussions of stellar densities, and the structural particles of stellar matter can be treated as massive charged points.


Ralph H. Fowler
(1889 –1944)

Chandrasekhar limit



$$p_{\mathrm{F}} = \left(3\pi^2\right)^{1/3} \hbar \cdot n^{1/3} \sim \hbar / r_{\mathrm{i}}$$

$$\frac{p_{\mathrm{F}}^2}{2m_{\mathrm{e}}} \sim \frac{GMm_{\mathrm{p}}}{R}, \quad R \sim \left(\frac{M}{m_{\mathrm{p}}}\right)^{1/3} r_{\mathrm{i}} \Longrightarrow R \propto M^{-1/3}$$

$$p_{\mathrm{F}}c \sim \frac{GMm_{\mathrm{p}}}{R}, \quad R \sim \left(\frac{M}{m_{\mathrm{p}}}\right)^{1/3} r_{\mathrm{i}} \Longrightarrow M_{\mathrm{Ch}} \sim \left(\frac{Gm_{\mathrm{p}}^2}{\hbar c}\right)^{-3/2} m_{\mathrm{p}}$$

More detailed calculation: $M_{\mathrm{Ch}} = 1.457 \left(\frac{2}{\mu_{\mathrm{e}}}\right)^2 M_{\odot}$

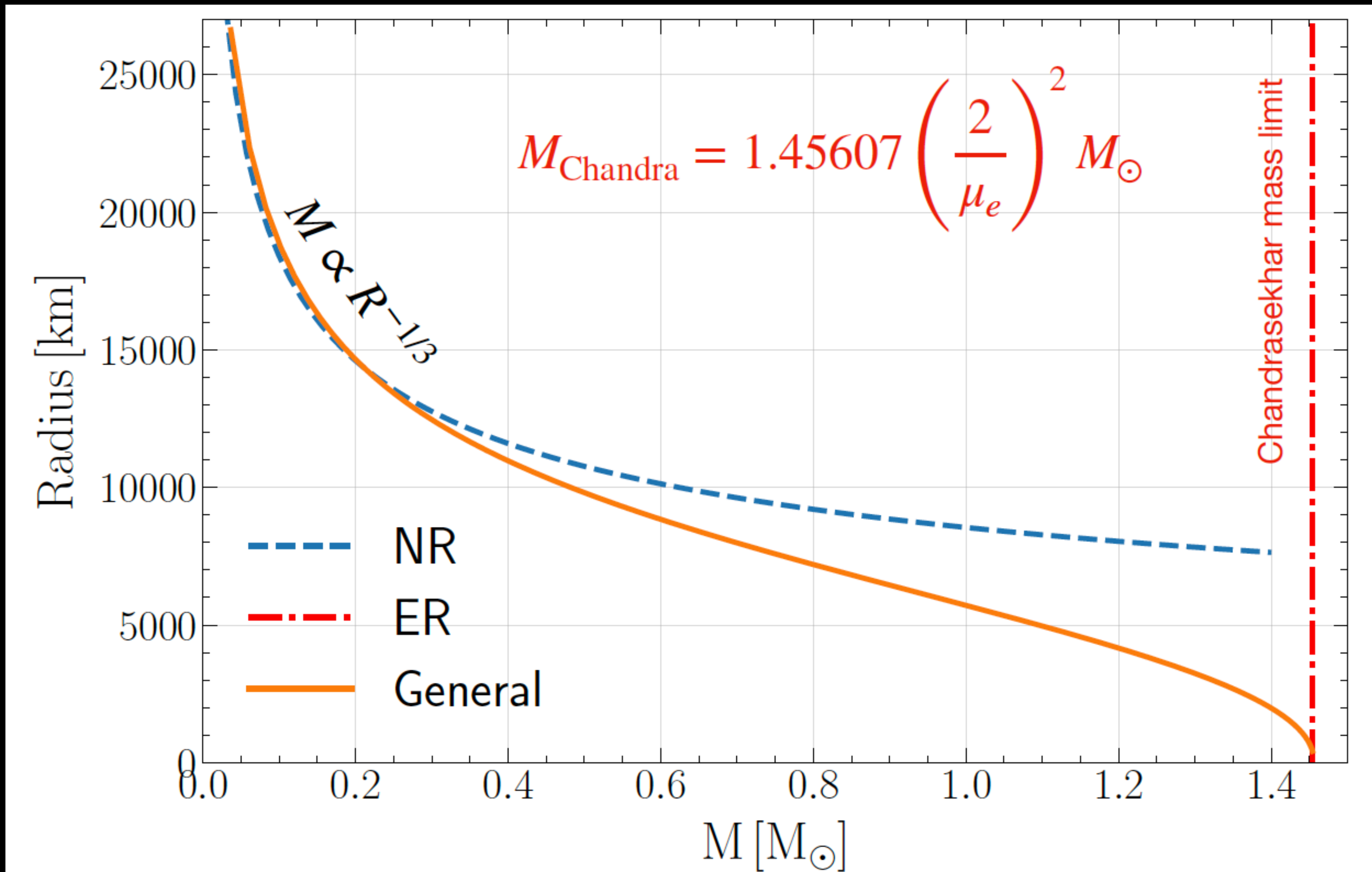
THE MAXIMUM MASS OF IDEAL WHITE DWARFS

By S. CHANDRASEKHAR

ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star* built on this model. This mass (=0.91⊙) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

it



$\cdot\, n^{1/3} \sim \hbar / r_{\rm i}$

M … $n_{\rm p}$ … $)^{1/3}$ $r_{\rm i} \Longrightarrow R \propto M^{-1/3}$

3

$r_{\rm i} \Longrightarrow M_{\rm Ch} \sim \left(\frac{G m_{\rm p}^2}{\hbar c}\right)^{-3/2} m_{\rm p}$

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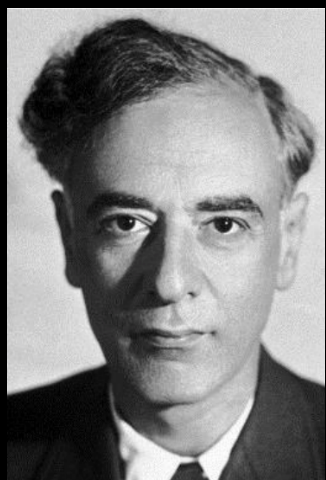
Chandrasekhar

After astronomers found stars with $\rho \sim 10^6\,\mathrm{g\,cm^{-3}}$



Landau 1932, before the discovery of neutron

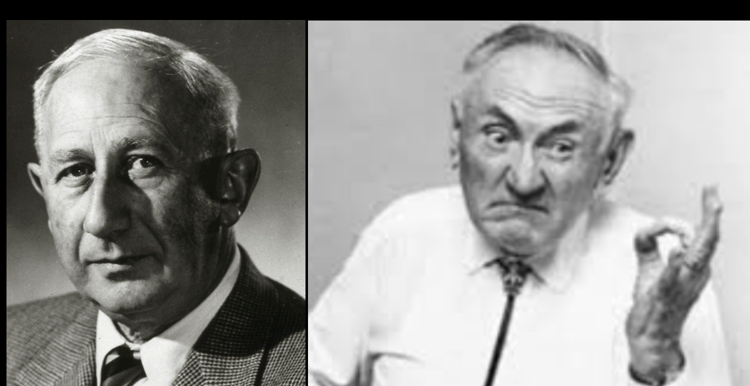
"We expect that this must occur when the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus".
"The laws of ordinary quantum mechanics break down. . . ".

The Existence of a Neutron.

By J. Chadwick, F.R.S.

(Received May 10, 1932.)

Chadwick, 1932

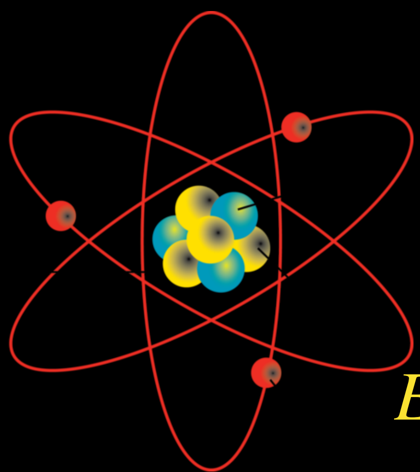


Baade & Zwicky, 1934

"With all reserve, we advance the view that supernovae represent the transition from ordinary stars into neutron stars, which in their final stages consist of closely packed neutrons."
"Neutrons are produced on the surface of an ordinary star [under the effect of cosmic rays] and " 'rain' down towards the center as we assume that the light pressure on neutrons is practically zero".

Why neutron stars?

Normal baryonic matter (Stars)

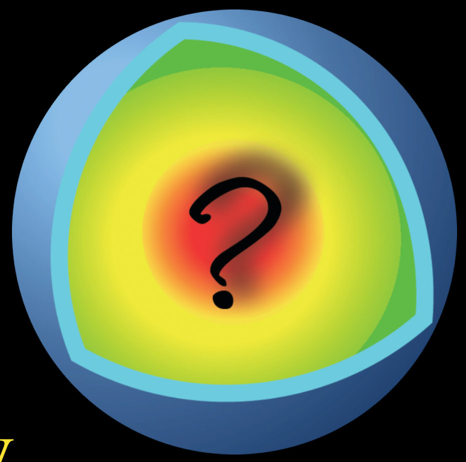


Compression

$p_F \propto n_e^{1/3}$

$E_e \sim 300 - 500\,\mathrm{MeV}$

Supranuclear matter



$\sim 20\,\mathrm{km}$

How to kill the energetic electrons?

$$e + p \rightarrow n + \nu_e$$

$$\mu_n = \mu_e + \mu_p$$

β equilibrium

The fraction of proton and electron

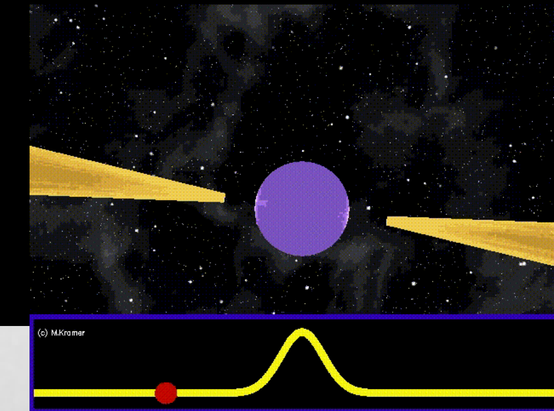
β equilibrium implies: $\frac{p_{F,n}^2}{2m_u} = \frac{p_{F,p}^2}{2m_u} + p_{F,e}c$

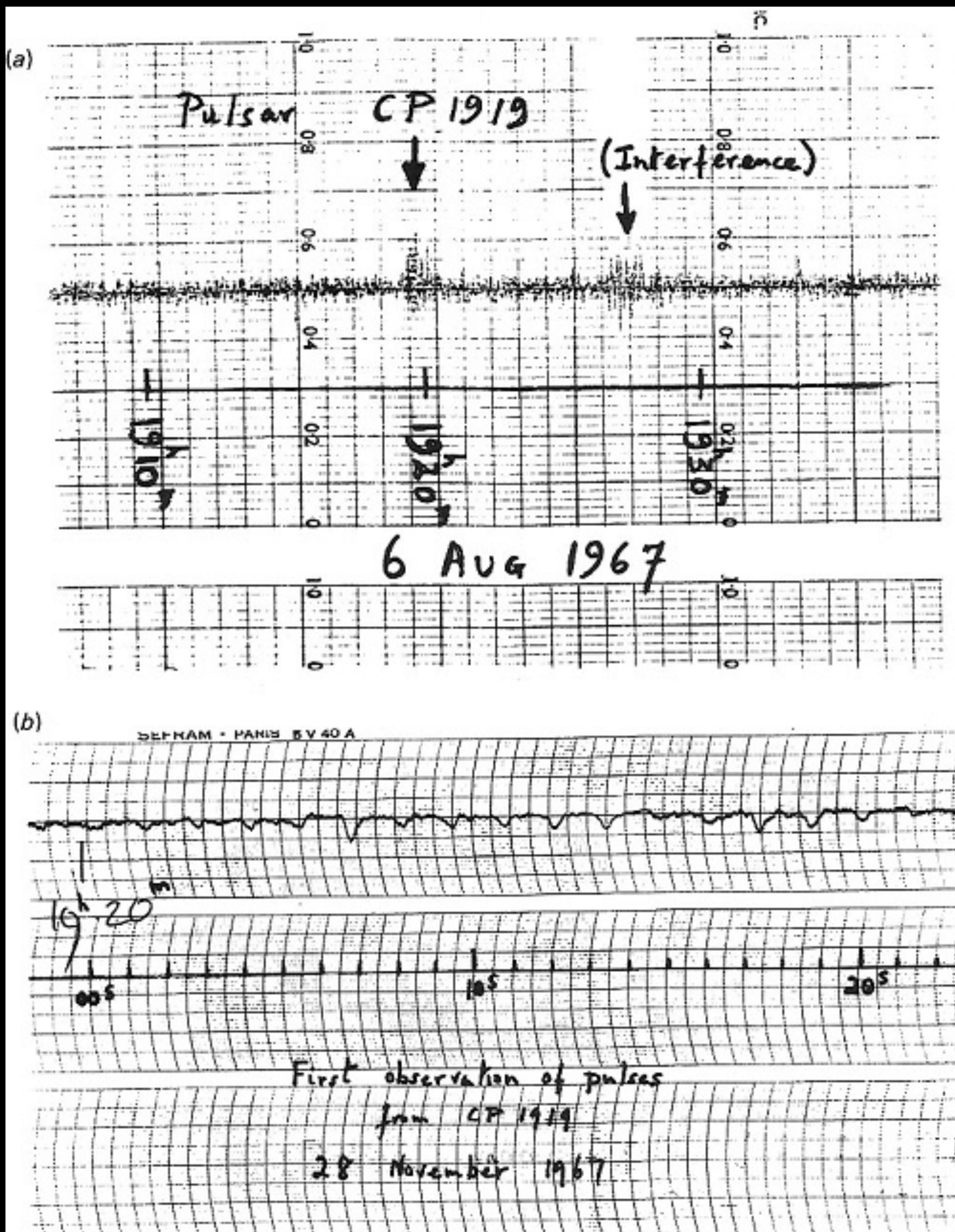
Charge neutrality: $n_e = n_p$

Homework: why we call it neutron star?

Assume that proton fraction is much smaller than the neutron fraction ($n_p \ll n_n$)

Discovery of pulsars





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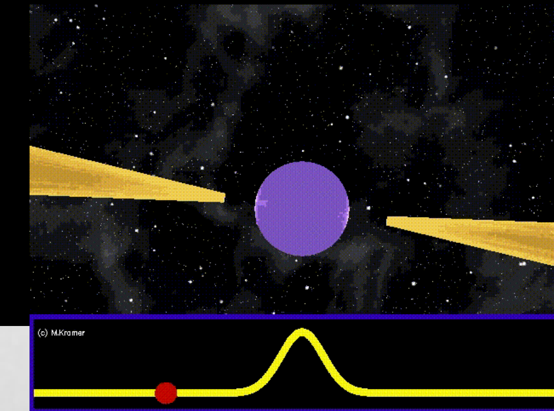
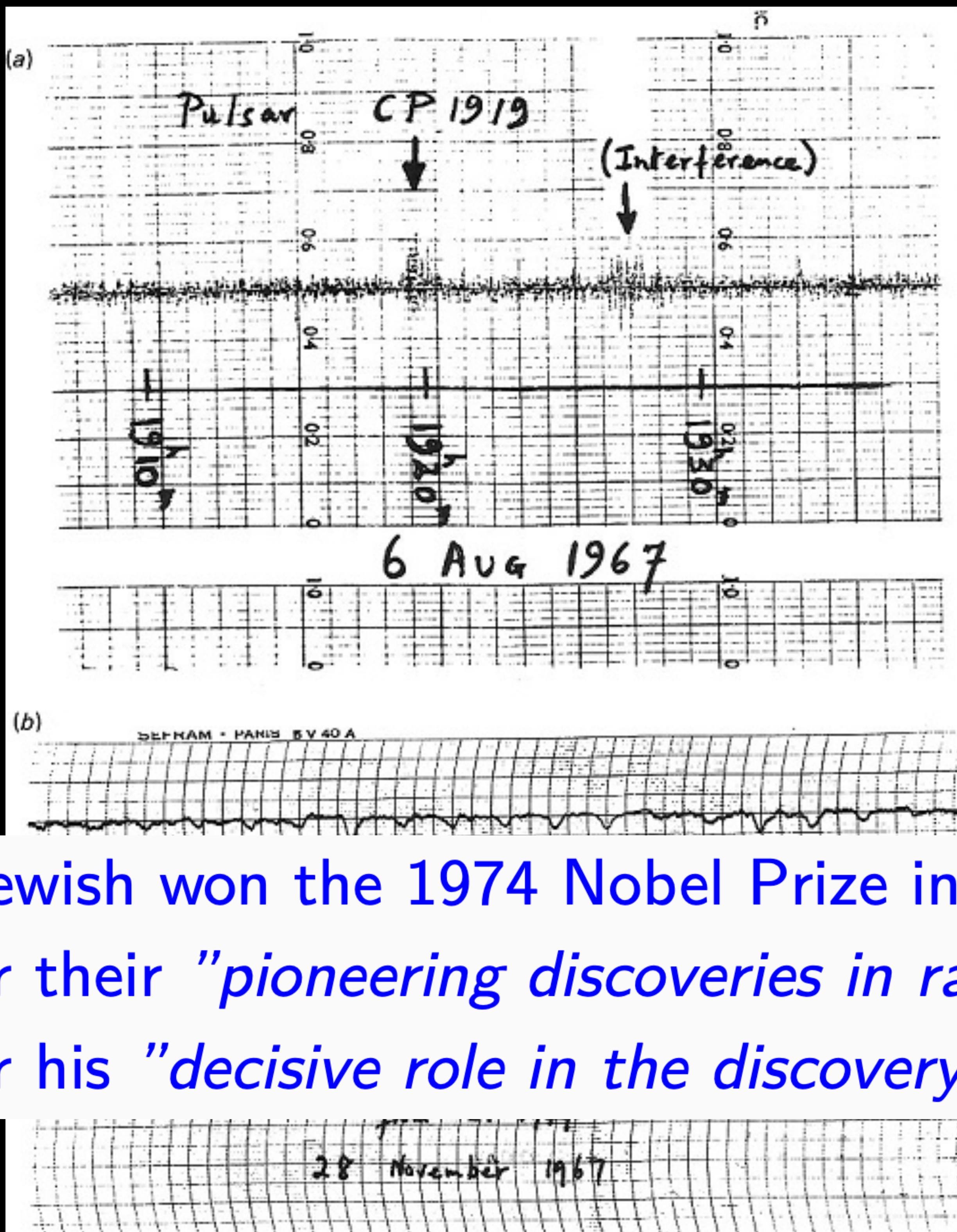




Jocelyn Bell Burnell

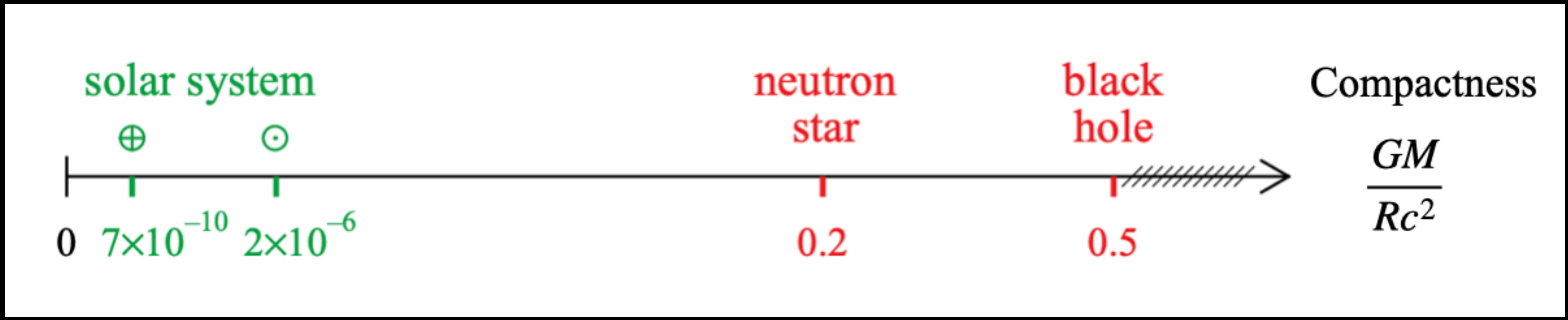
Hewish won the 1974 Nobel Prize in Physics along with Sir Martin Ryle for their *"pioneering discoveries in radio astrophysics."* Hewish was cited for his *"decisive role in the discovery of pulsars."*

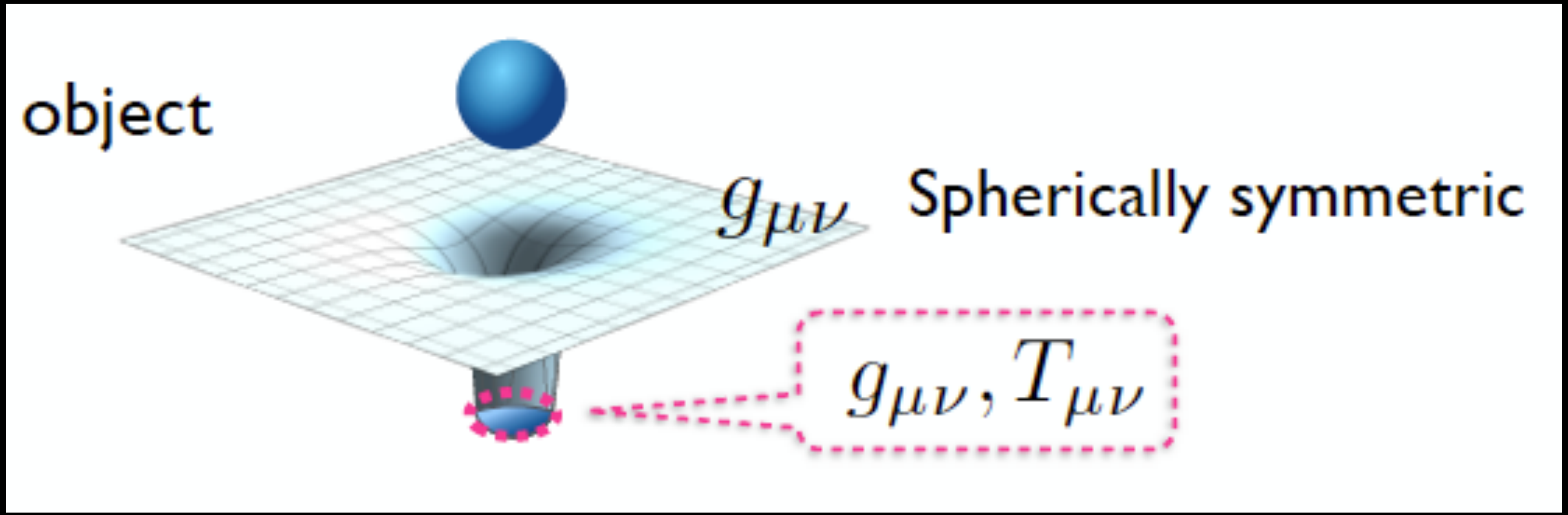
Neutron star structures

To give structures (mass and radius et al.) of neutron stars, we need:

(1). equation of state of the dense matter

(2). The hydro equilibrium equations in General Relativity





Matter field: thermodynamics and hydrodynamics

We restrict our attention to the case of a perfect fluid with equilibrium composition

$$\epsilon = \epsilon(\rho, s), \quad p = p(\rho, s)$$

First law of thermodynamics

$$\mathrm{d}\left(\frac{\varepsilon}{\rho}\right) = -p\,\mathrm{d}\left(\frac{1}{\rho}\right) + T\,\mathrm{d}s$$

$$p = \frac{-\partial(\varepsilon/\rho)}{\partial(1/\rho)} = \rho^2 \frac{\partial(\varepsilon/\rho)}{\partial\rho}, \quad T = \frac{\partial(\varepsilon/\rho)}{\partial s}$$

For mature NSs, $T \ll T_{\mathrm{F}}$

$$\epsilon = \epsilon(\rho), \quad p = p(\rho) \longrightarrow p = p(\epsilon)$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

For perfect fluid

$$T^{\alpha\beta} = (\epsilon + p) u^\alpha u^\beta + p g^{\alpha\beta}$$

The motion of the fluid element is governed by

$$\nabla_\alpha T^{\alpha\beta} = 0$$

And the conservation of the restmass

$$\nabla_\alpha \left(\rho u^\alpha\right) = 0$$

Tolman-Oppenheimer-Volkoff solution

Static Solutions of Einstein's Field Equations for Spheres of Fluid

RICHARD C. TOLMAN
Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California
(Received January 3, 1939)

A method is developed for treating Einstein's field equations, applied to static spheres of fluid, in such a manner as to provide explicit solutions in terms of known analytic functions. A number of new solutions are thus obtained, and the properties of three of the new solutions are examined in detail. It is hoped that the investigation may be of some help in connection with studies of stellar structure. (See the accompanying article by Professor Oppenheimer and Mr. Volkoff.)

On Massive Neutron Cores

J. R. OPPENHEIMER AND G. M. VOLKOFF
Department of Physics, University of California, Berkeley, California
(Received January 3, 1939)

It has been suggested that, when the pressure within stellar matter becomes high enough, a new phase consisting of neutrons will be formed. In this paper we study the gravitational equilibrium of masses of neutrons, using the equation of state for a cold Fermi gas, and general relativity. For masses under $\frac{1}{3}\odot$ only one equilibrium solution exists, which is approximately

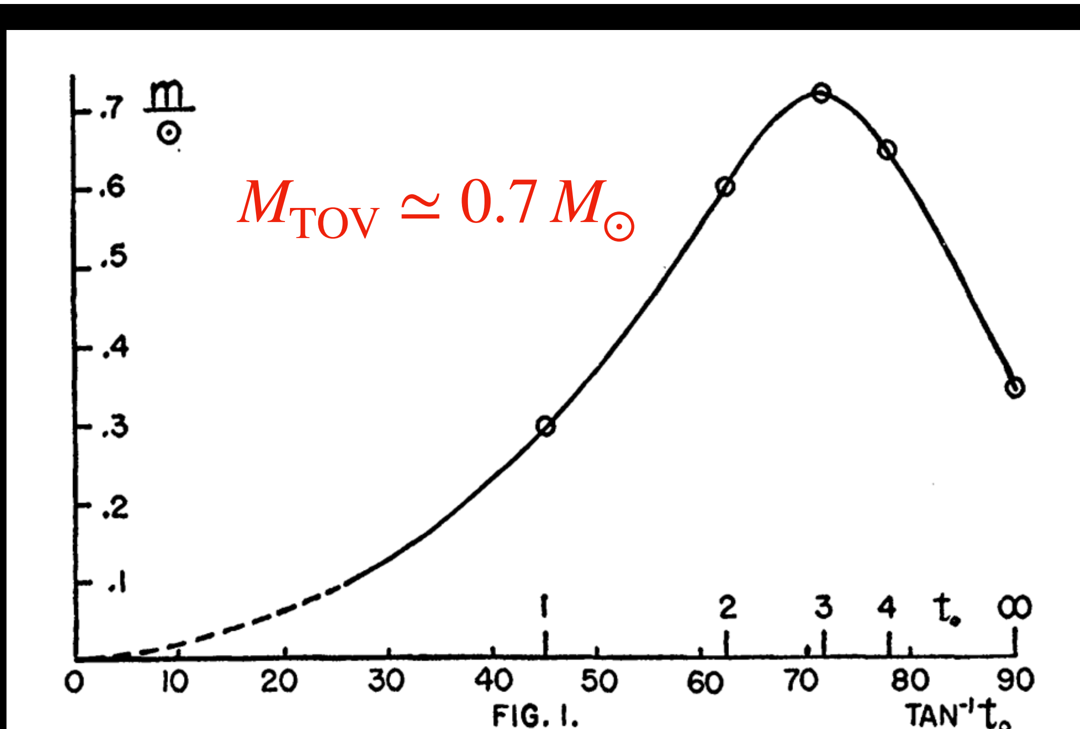


FIG. 1. Dependence of m on t_0 for neutrons.

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\epsilon}{r^2}\left(1+\frac{P}{\epsilon c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon$$

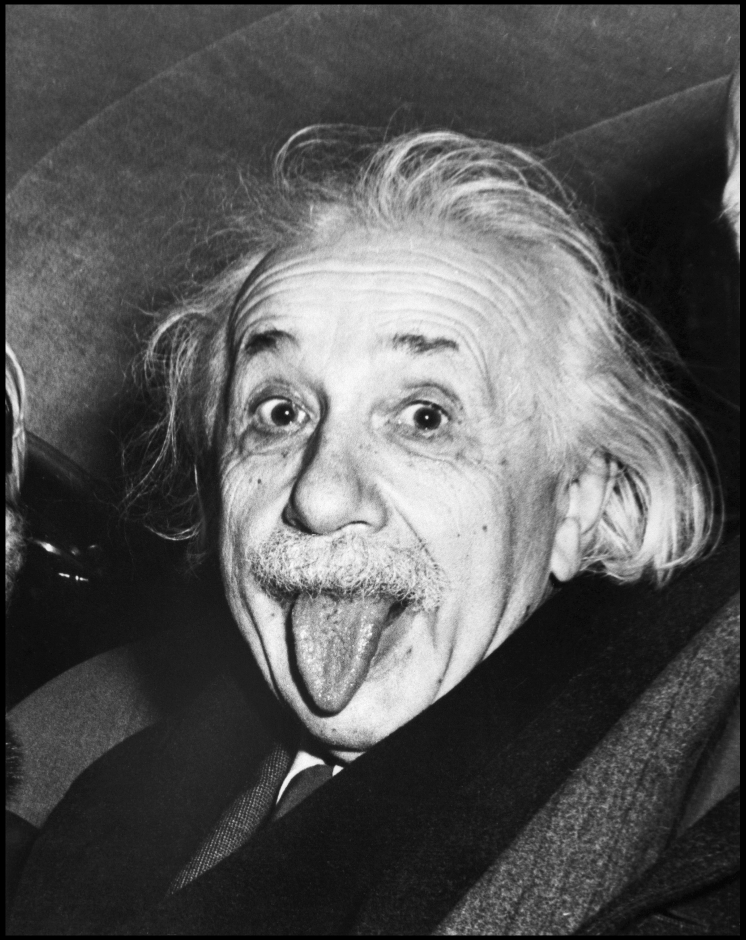
$$p = p(\epsilon)$$

Homework I: try to write a TOV solver, and reproduce the results of Oppenheimer & Volkoff 1939

EOS: Non-relativistic free neutron gas

Why there is a maximal mass?

General relativity!



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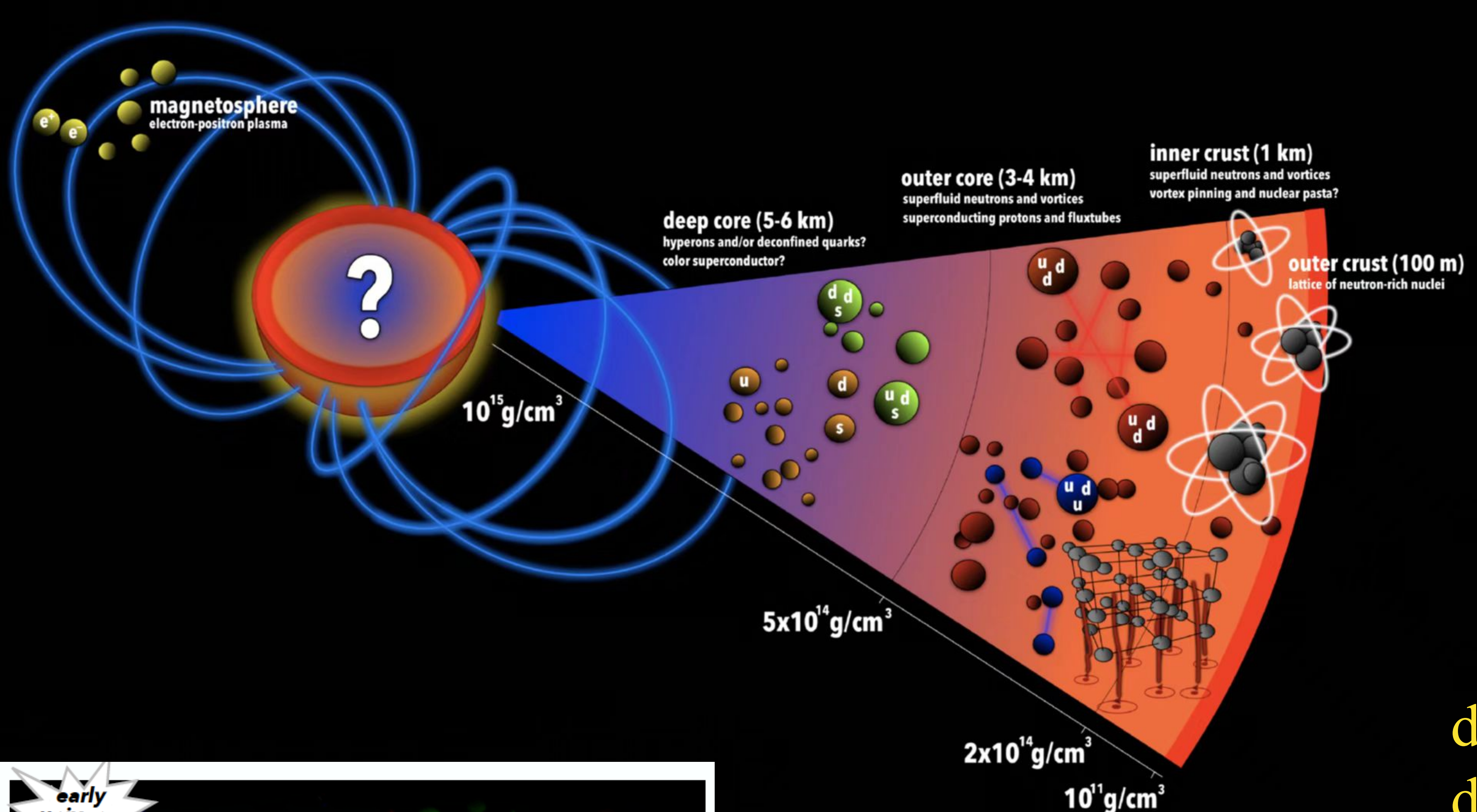
Newtonian gravity: pressure balances gravity

General relativity: pressure also contributes to gravity

The picture of a neutron star as a self-gravitating ball of non-interacting fermions is of course simplified!



$\sim 10^6$ g/cm^3

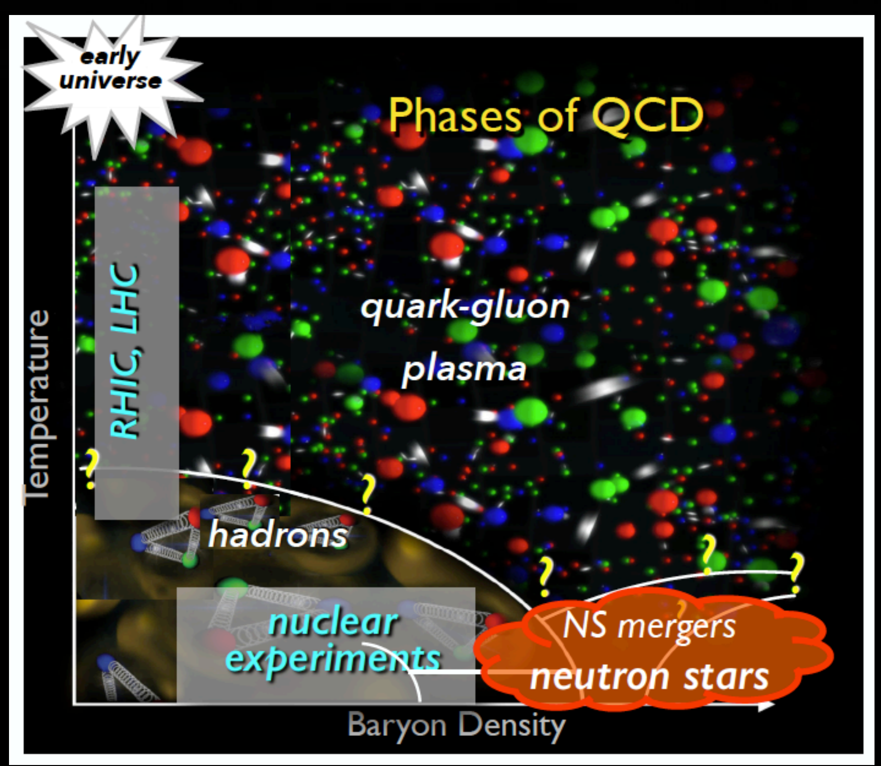
inverse β-decay

$\sim 10^{11}$ g/cm^3

neutron drip

$\sim$ few $\times 10^{14}$ g/cm^3

deep core ≳2 x nuclear density, nucleons overlap - new degrees of freedom relevant??



Composition and state of matter (EOS)

We don't know, because non-perturbative QCD of many body problem

The crust of neutron stars

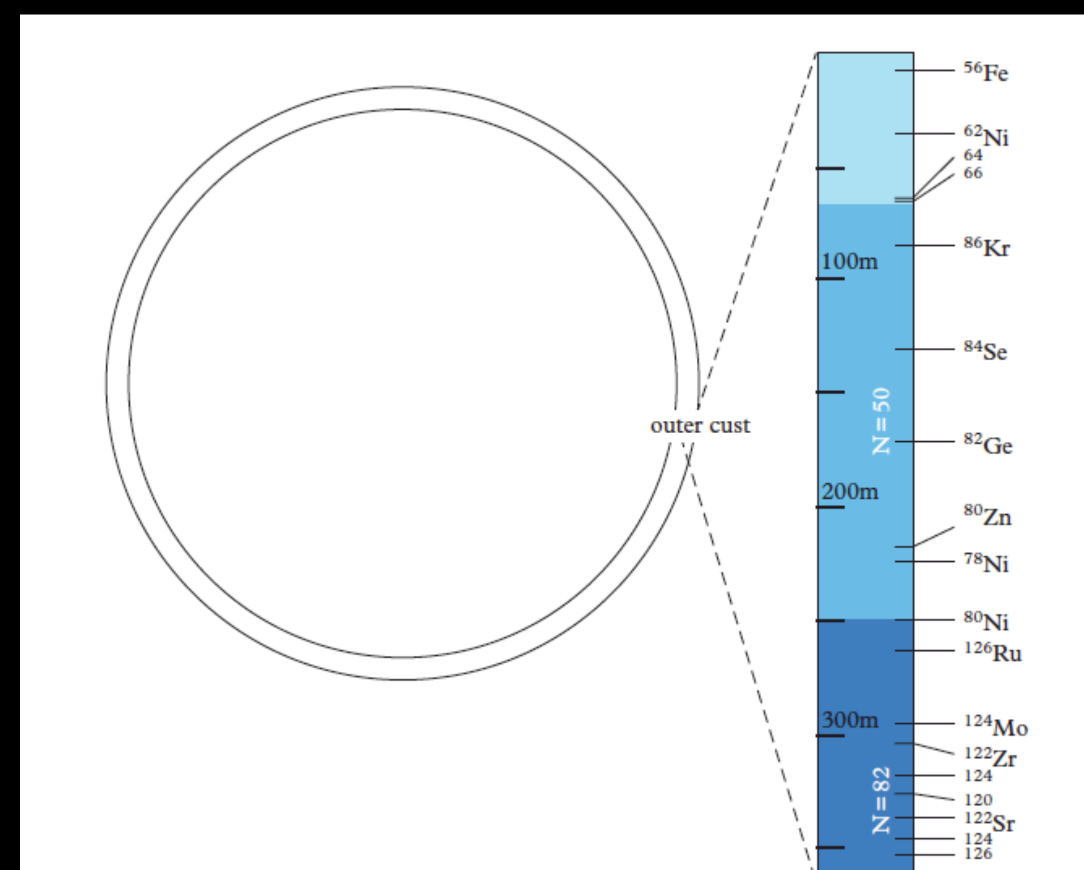
Solid? liquid? or gas? **The electrons are degenerate, but what about the ions? The ions are non-degenerate with energy set by the thermal energy** $\sim k_B T$

$$\Gamma = Z^2 e^2 / a_i k_B T$$

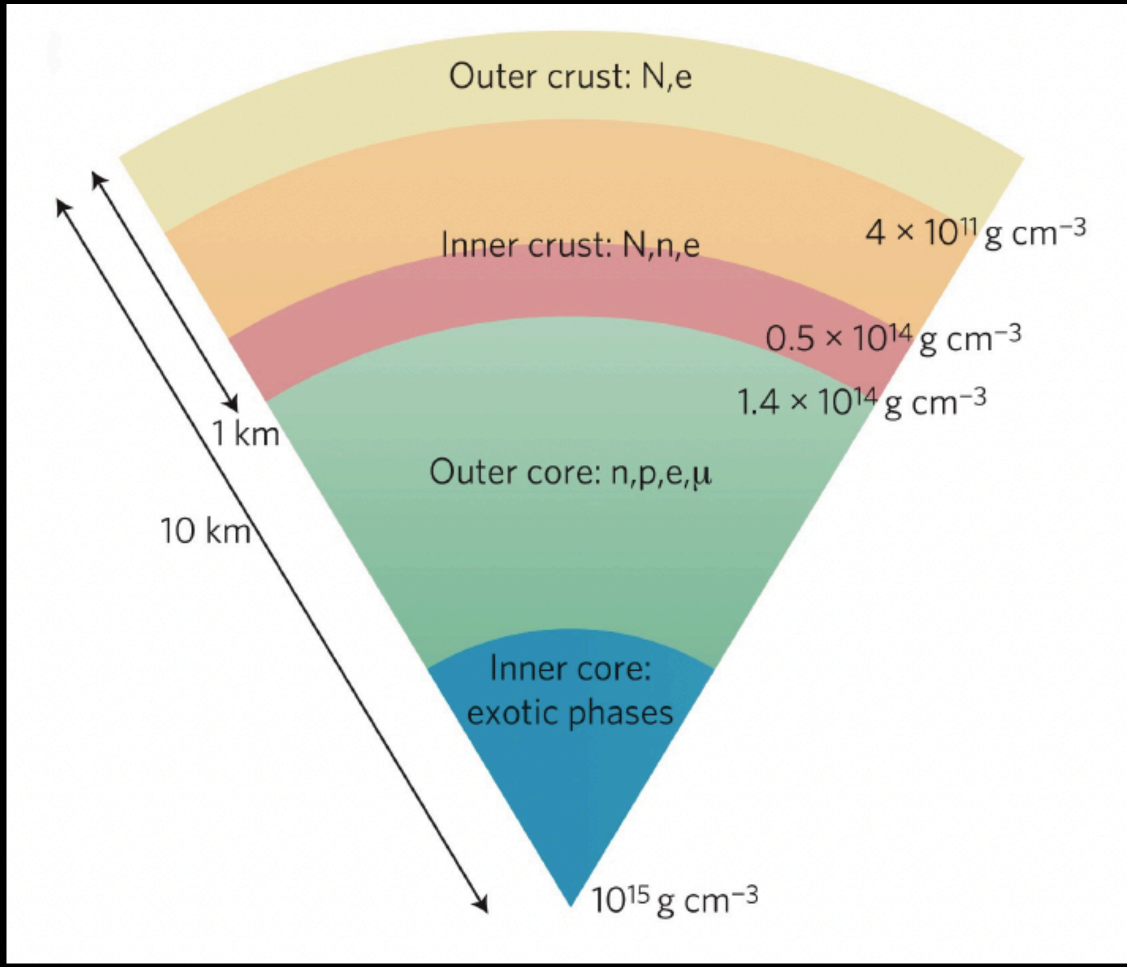
For $\Gamma > 175$, the Coulomb energy is strong enough to force the ions to fall into a lattice, giving a Coulomb solid

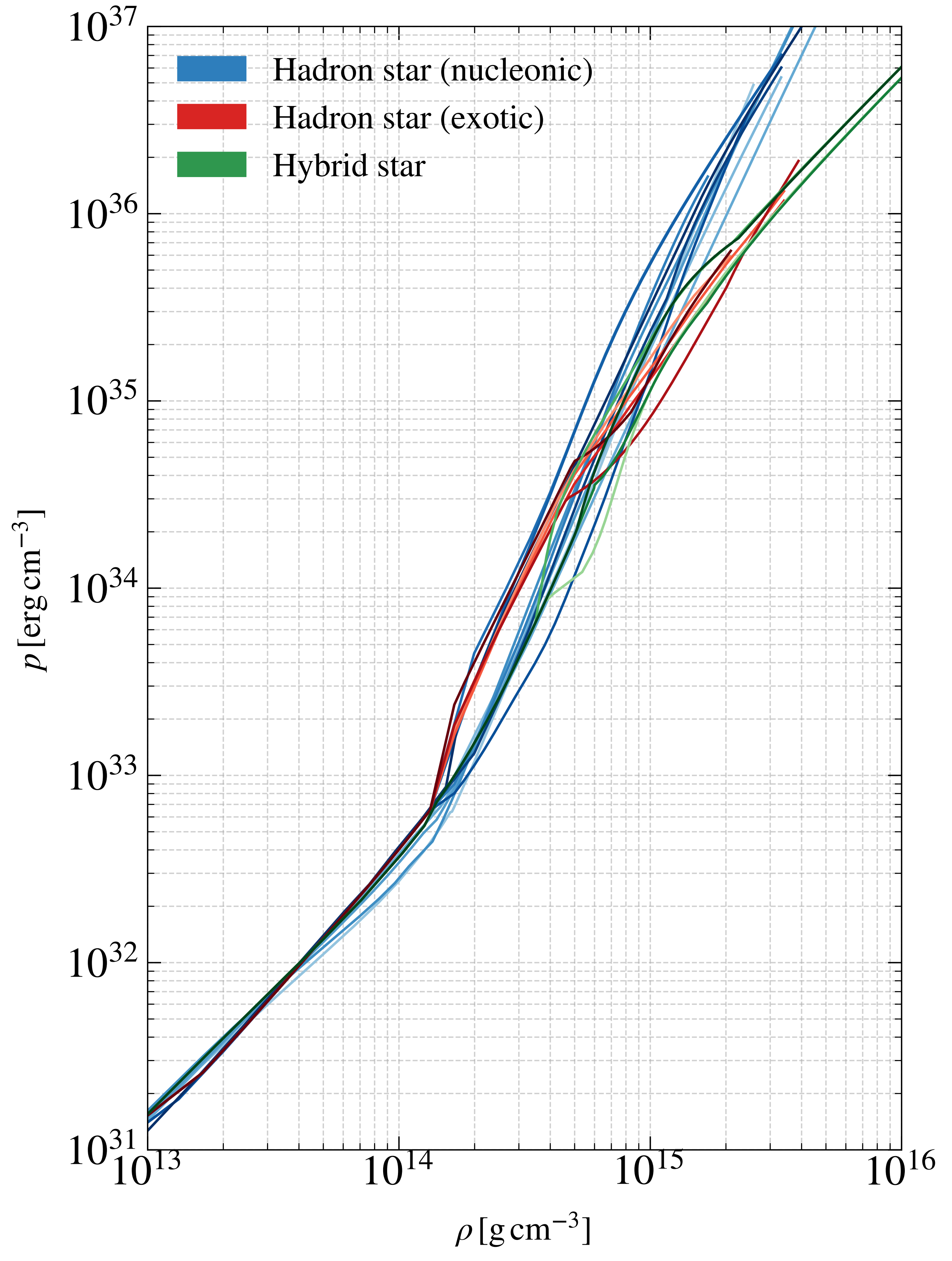
Electron capture.

$\rho_{\max}$ [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^6	^{56}Fe	26	30	1404.05
2.71×10^8	^{62}Ni	28	34	449.48
1.33×10^9	^{64}Ni	28	36	266.97
1.50×10^9	^{66}Ni	28	38	259.26
3.09×10^9	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92



EoS models—conventional NSs





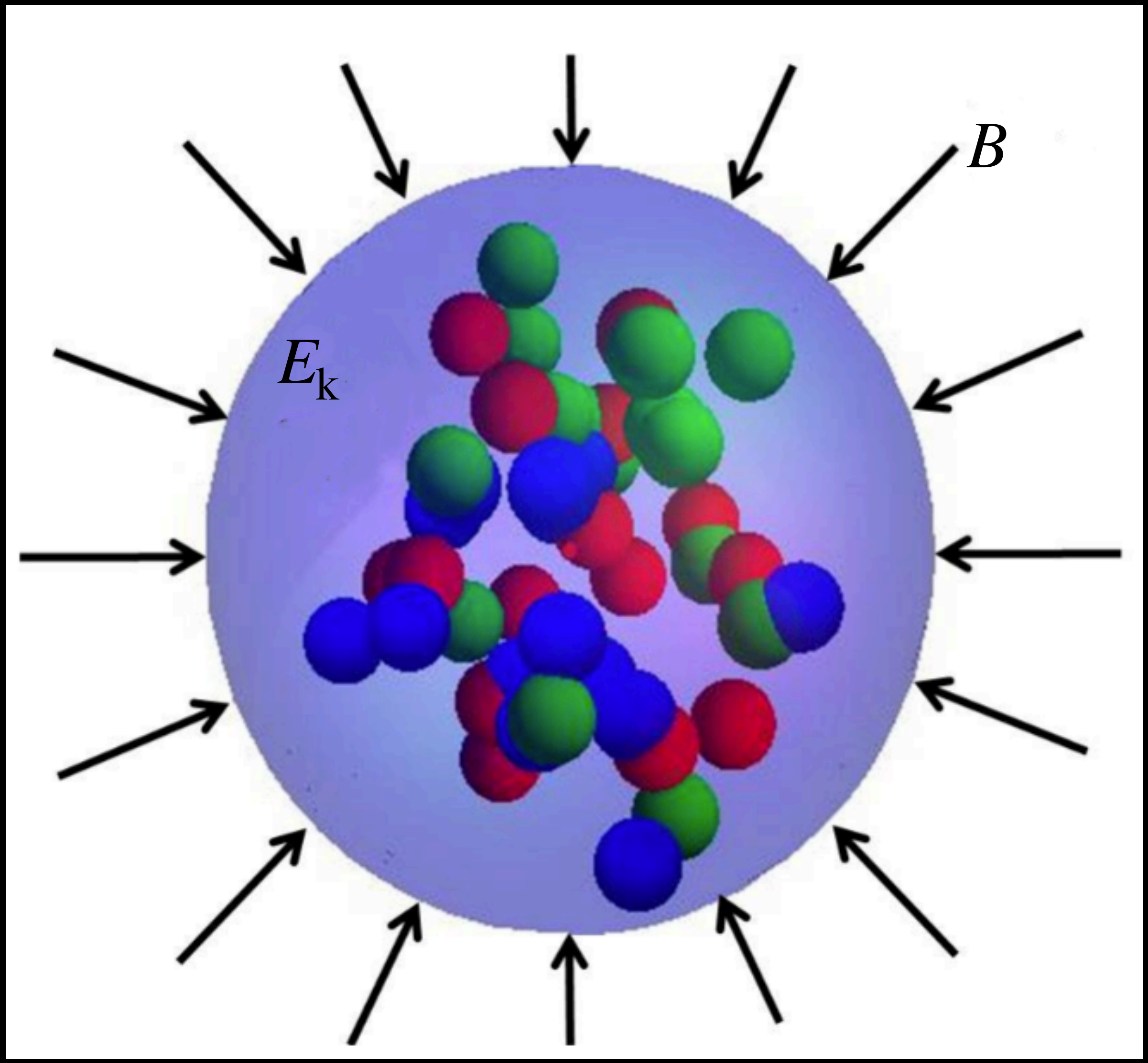
- Nucleon star: *npeμ* matter

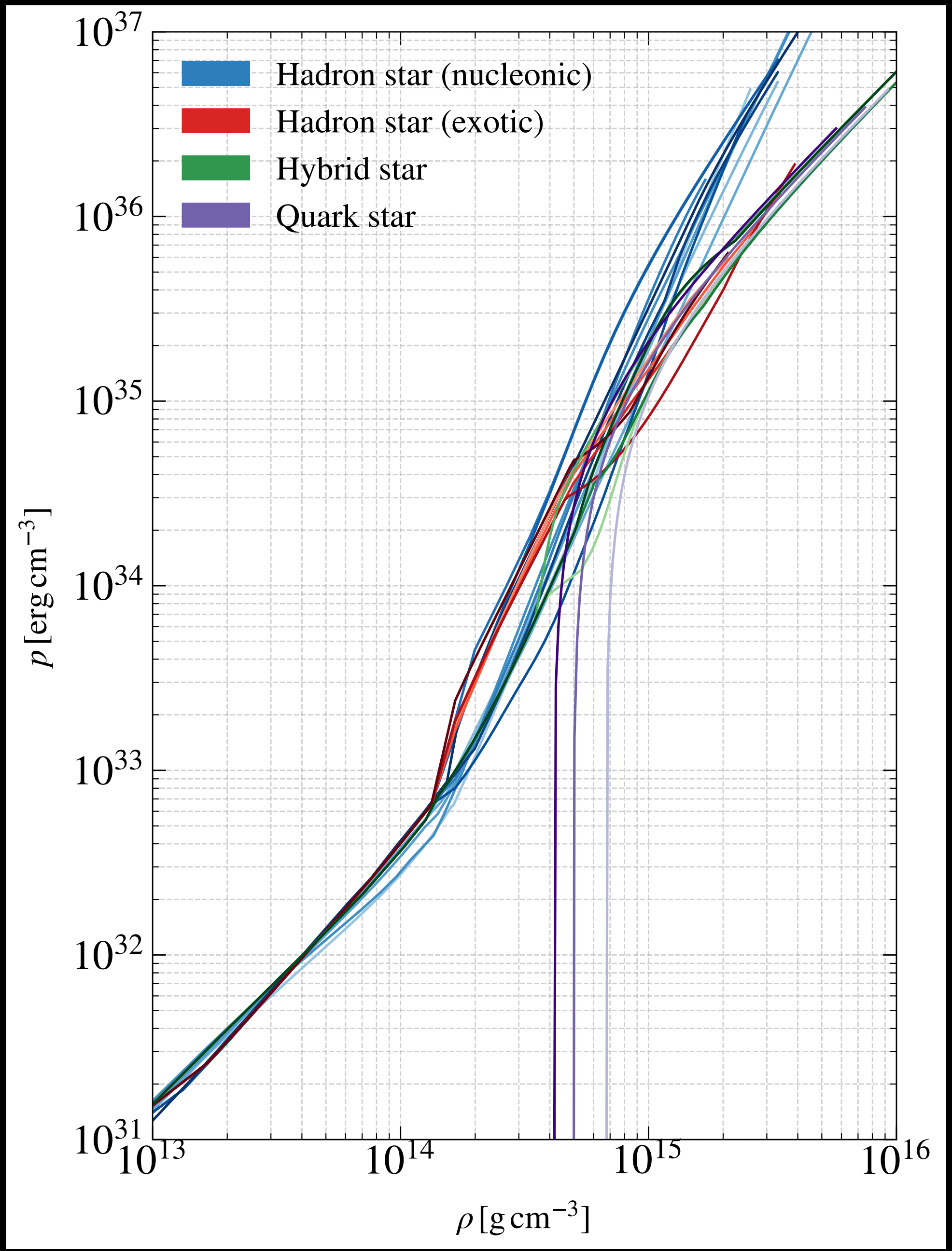
$\mu_e > m_\mu c^2 \approx 105\mathrm{MeV} \qquad e \longrightarrow \mu + \bar{\nu}_\mu + \nu_e$

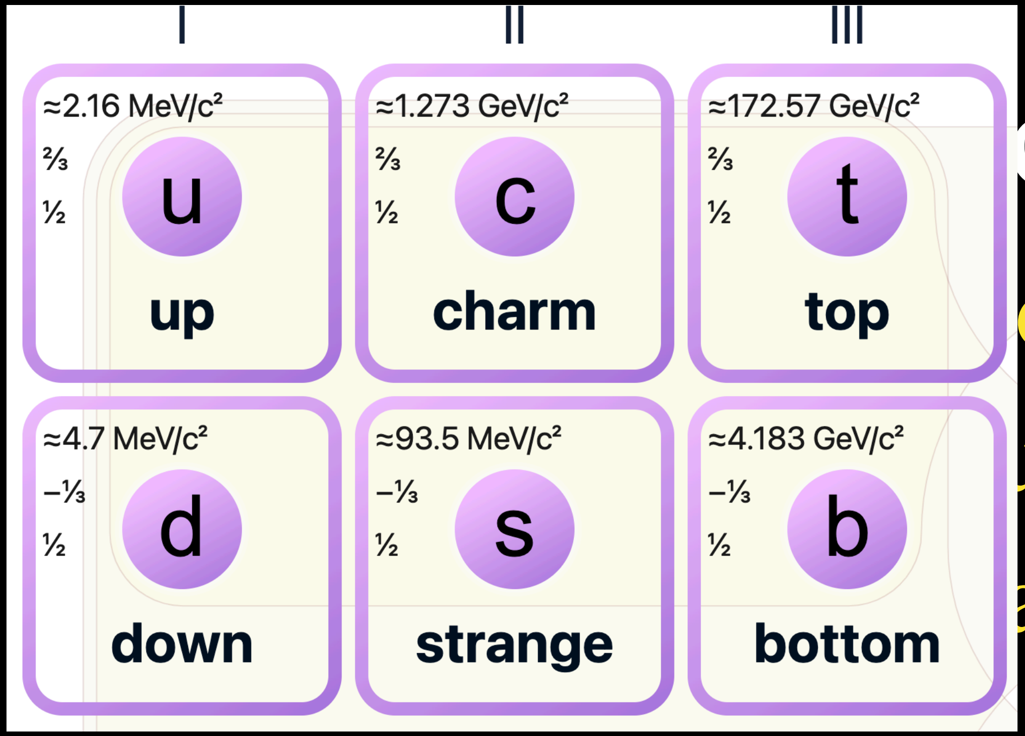
- New freedom in the inner core: hyperons? mesons? quarks?

EoS models—quark stars

Witten's conjecture: Quark matter composed of nearly equal number of u, d, s quarks could be the ground state of strong matter

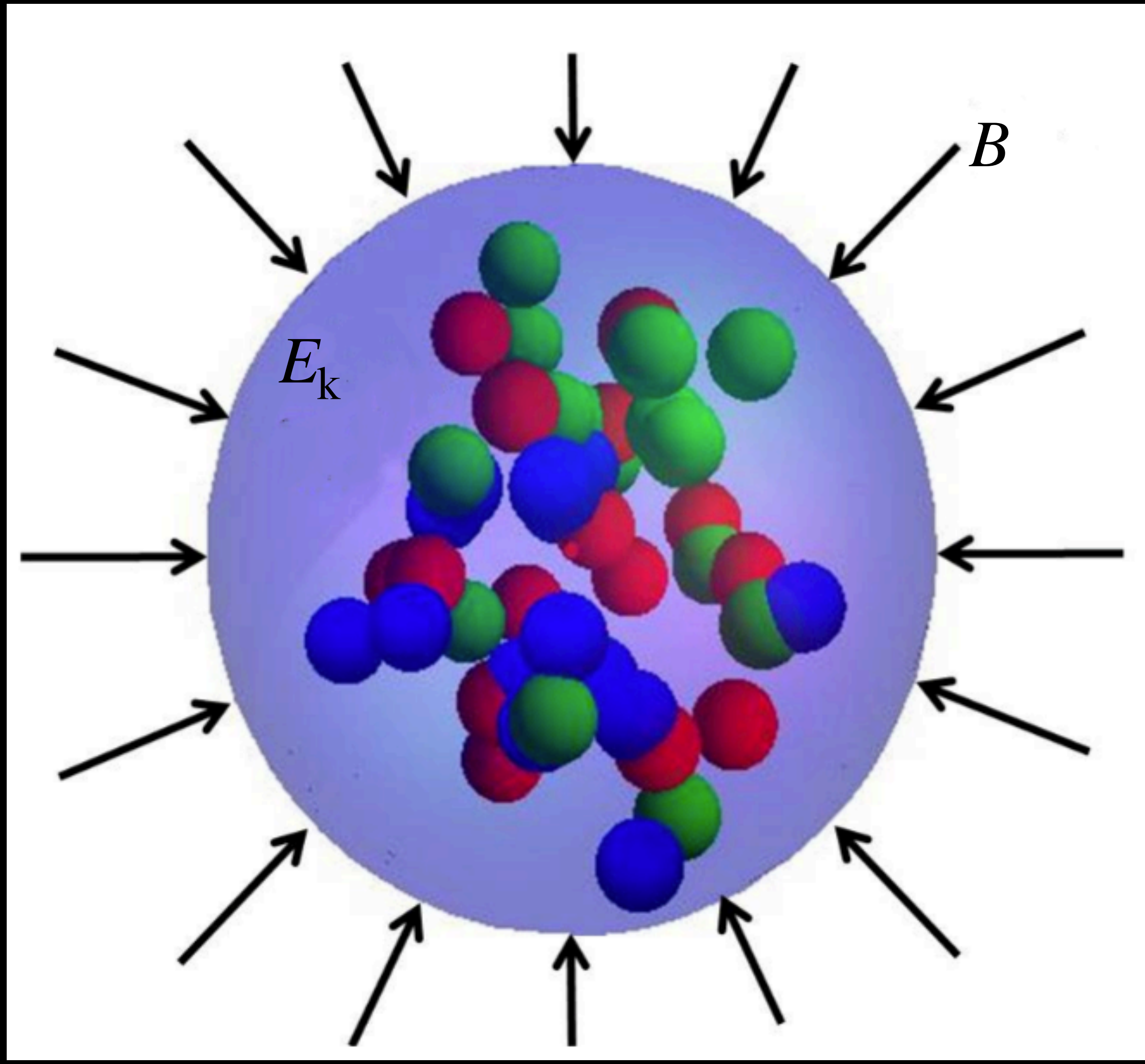


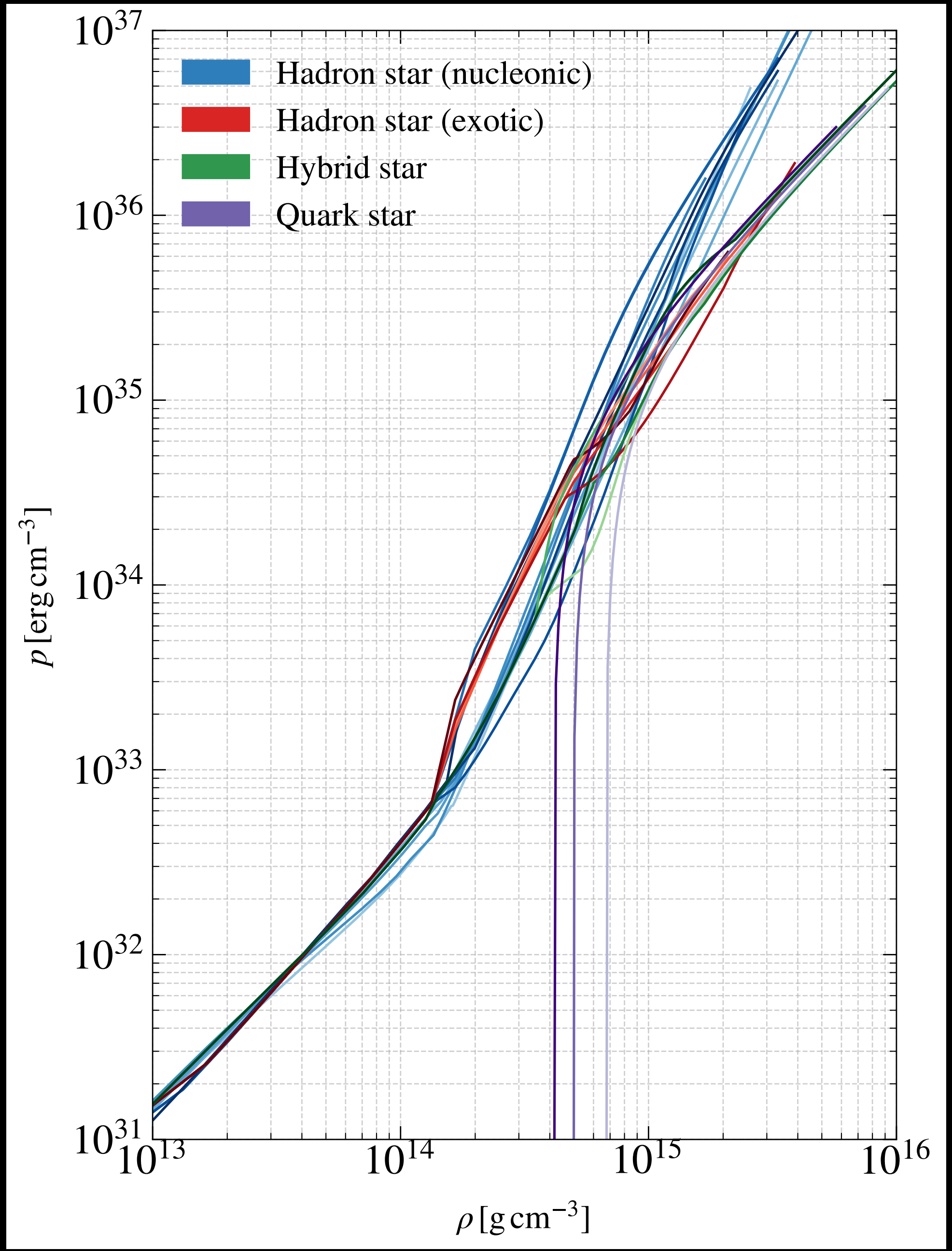




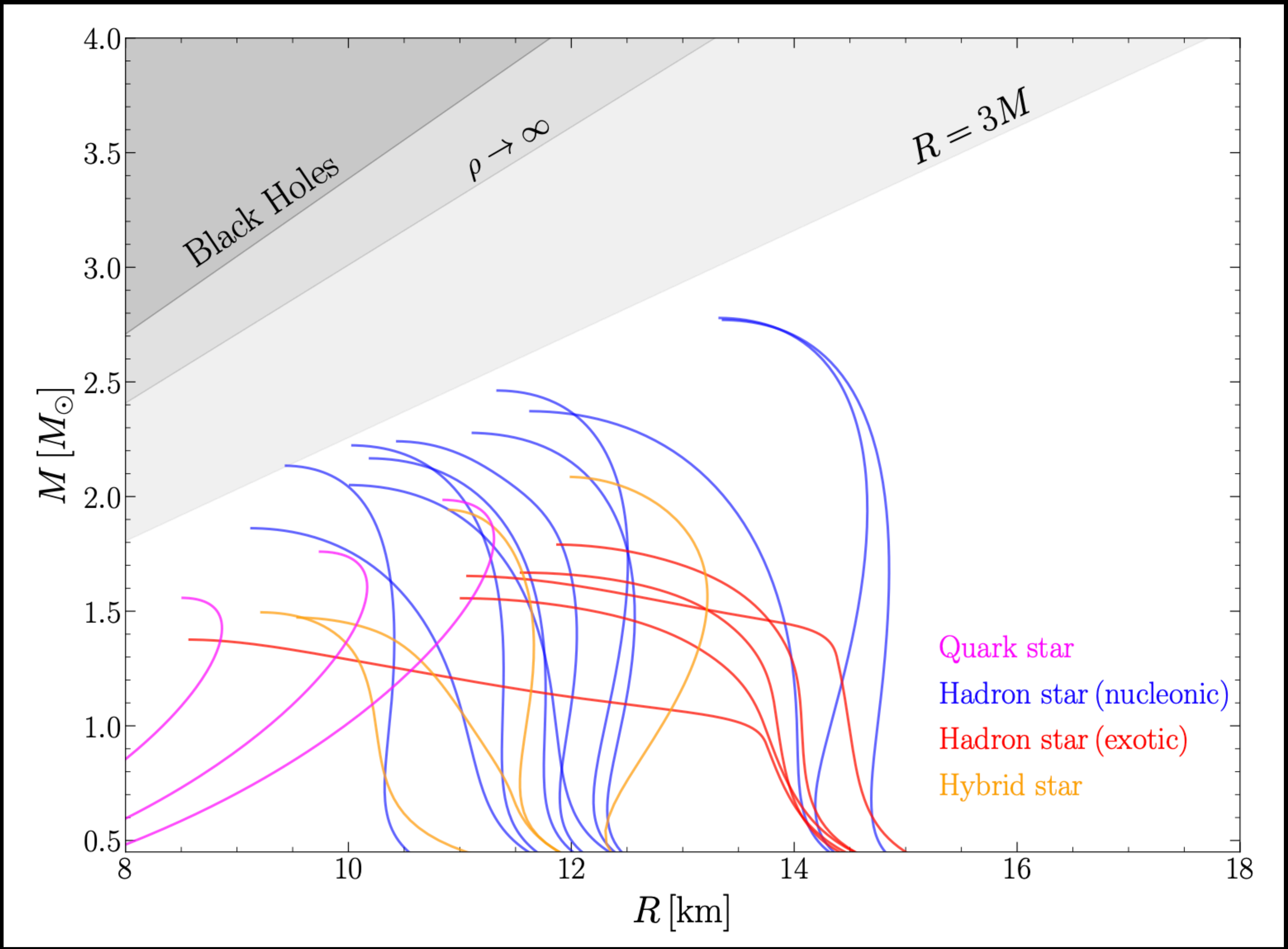
odels—quark stars

conjecture: Quark matter composed of ual number of *u*, *d*, *s* quarks could be the ate of strong matter



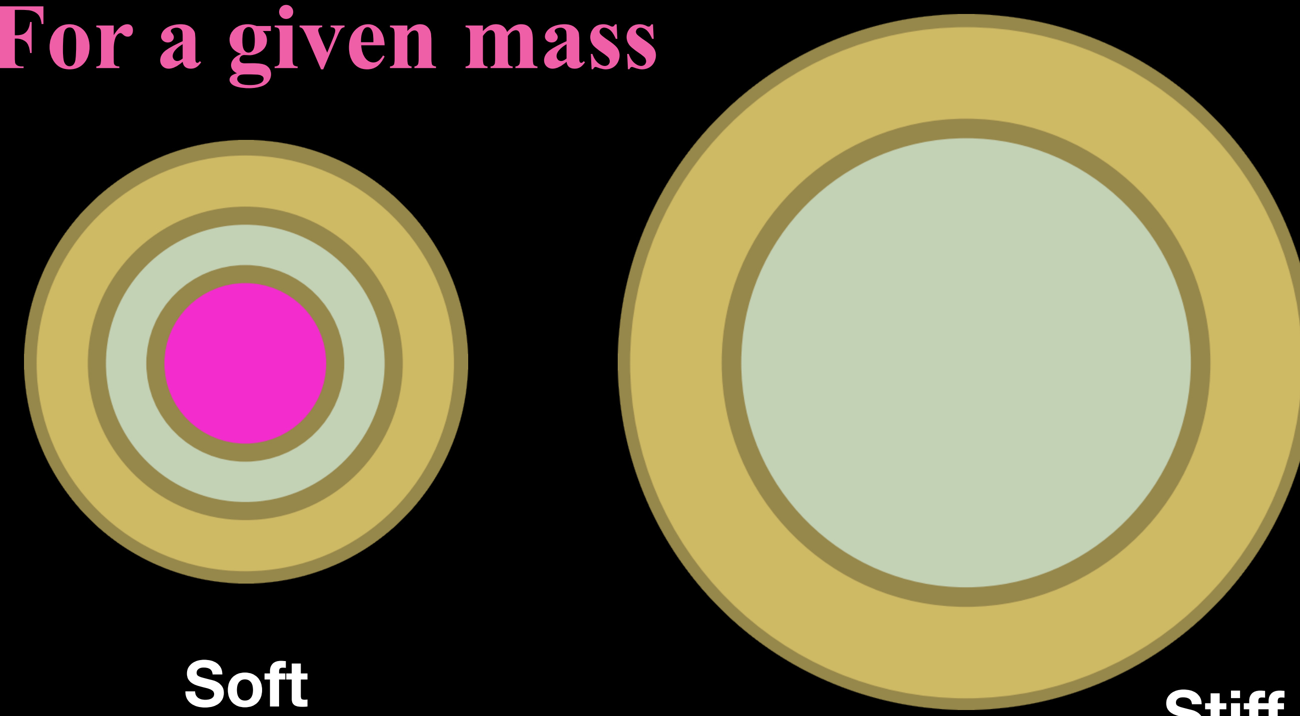


Mass-radius relation for neutron stars



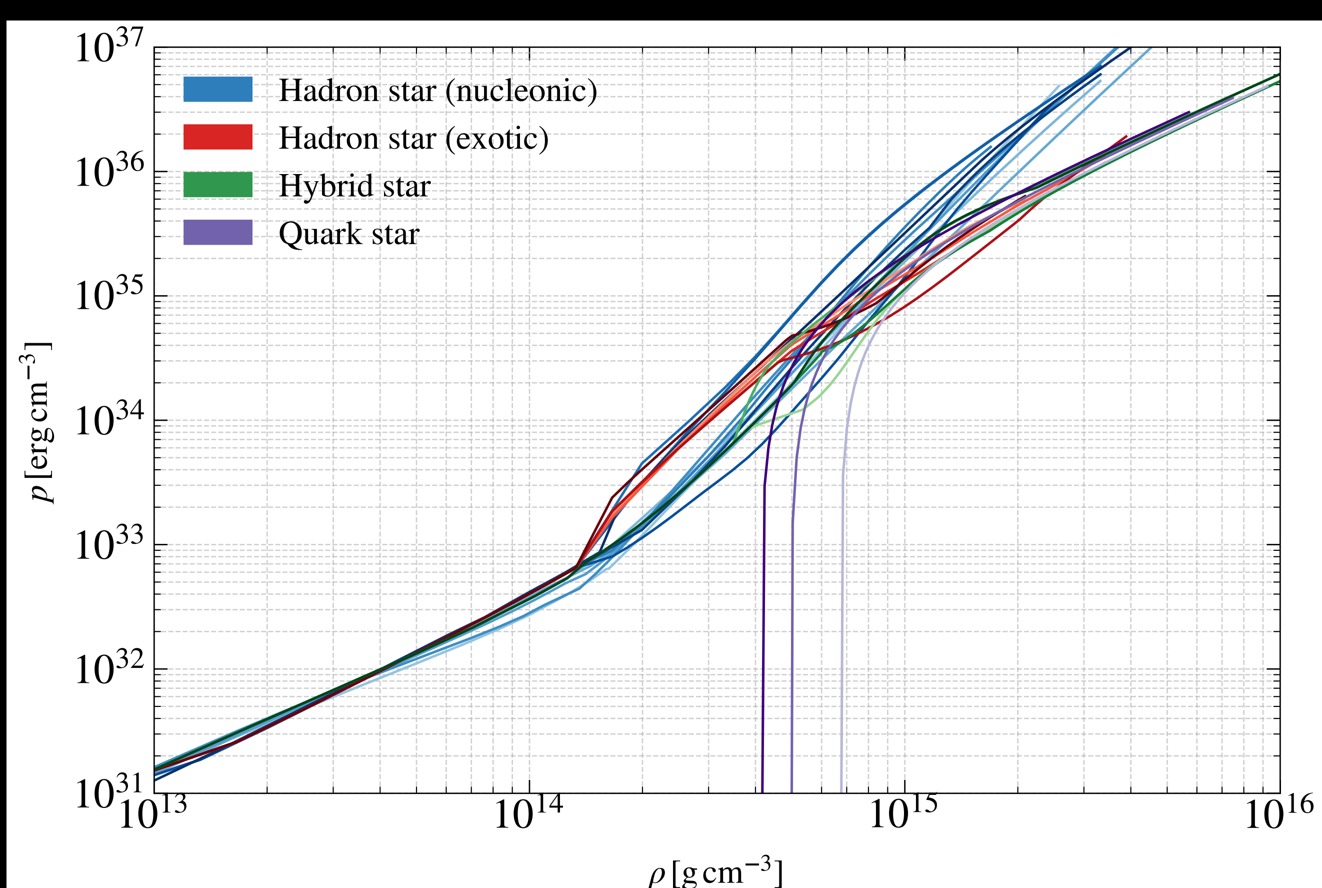
Stiff or soft? $P \sim \rho^{\Gamma}$

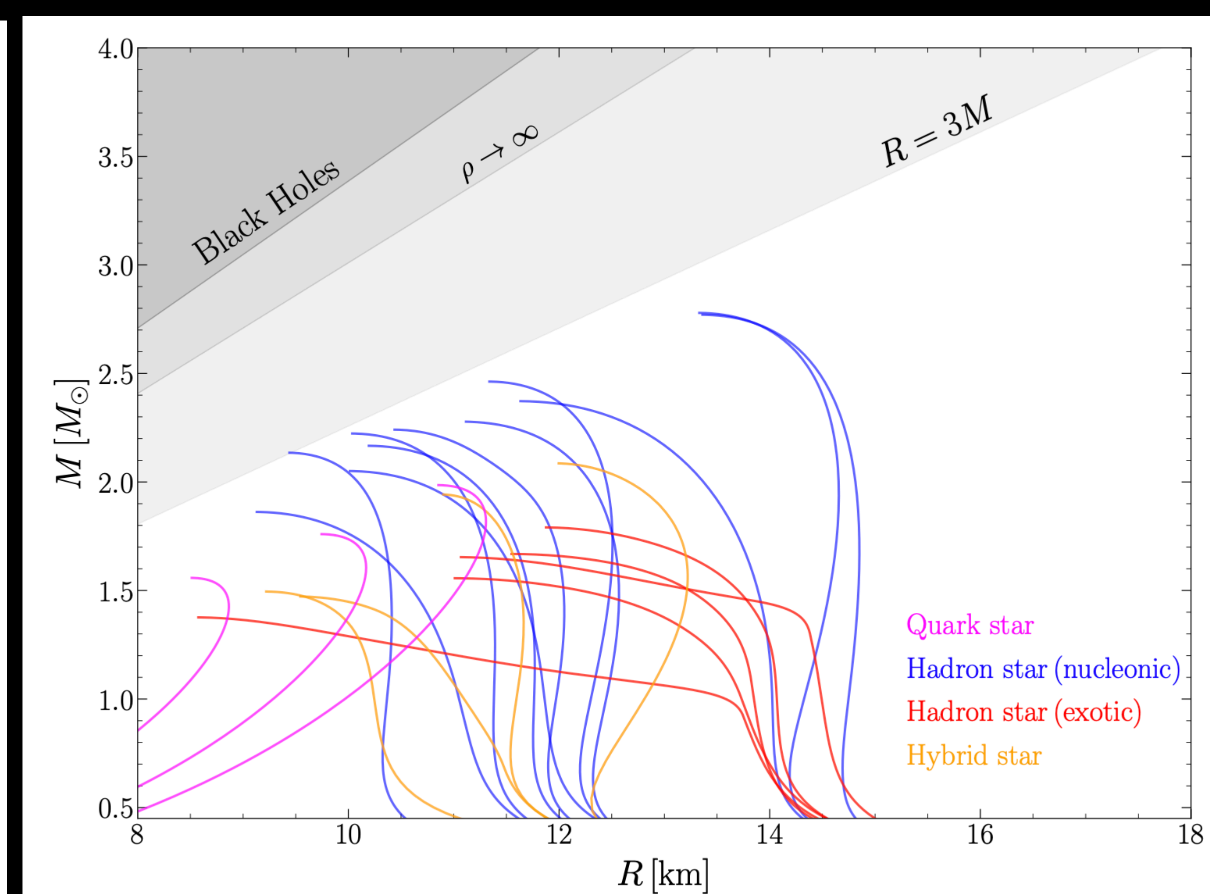
For a given mass



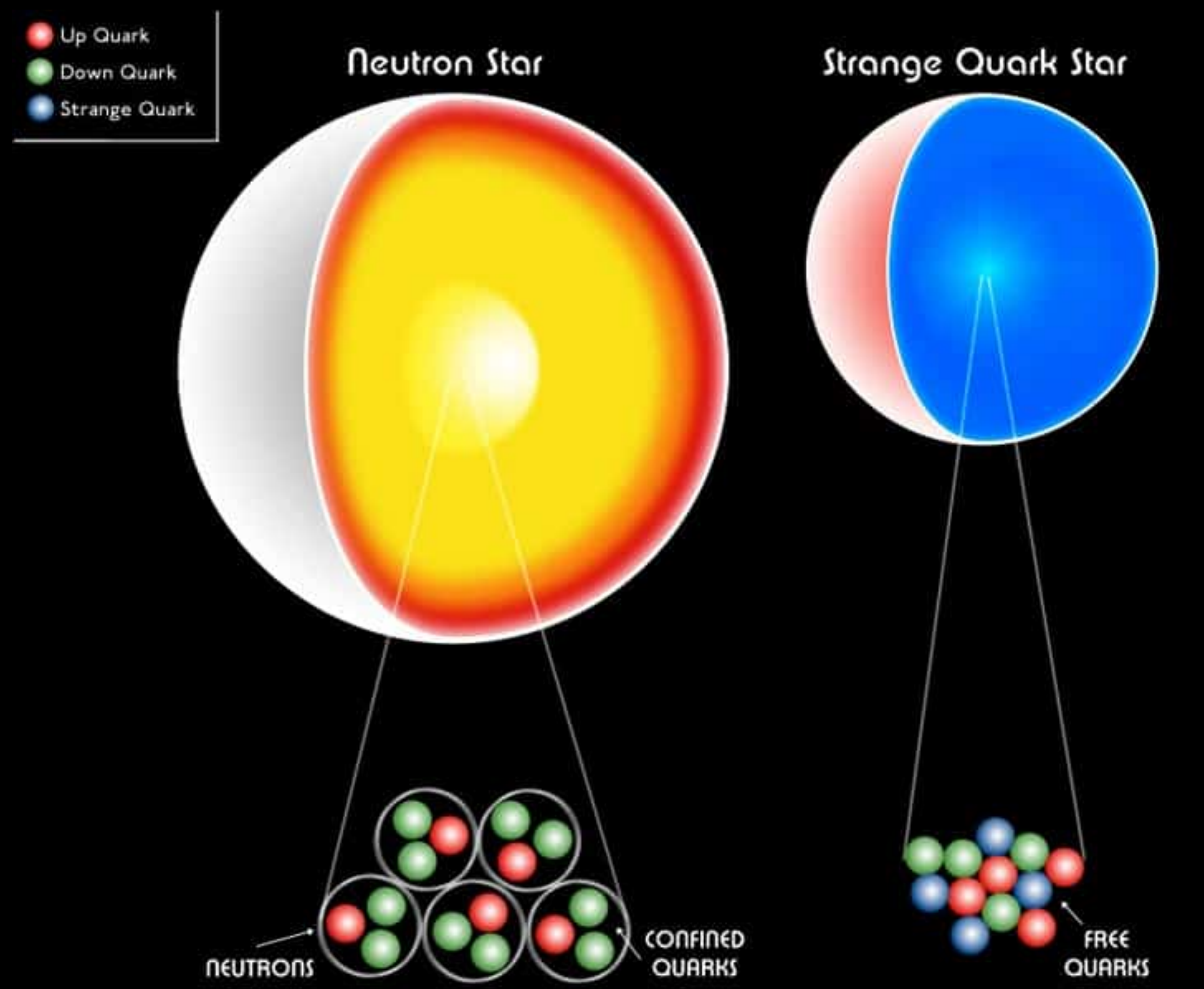
For different EoSs

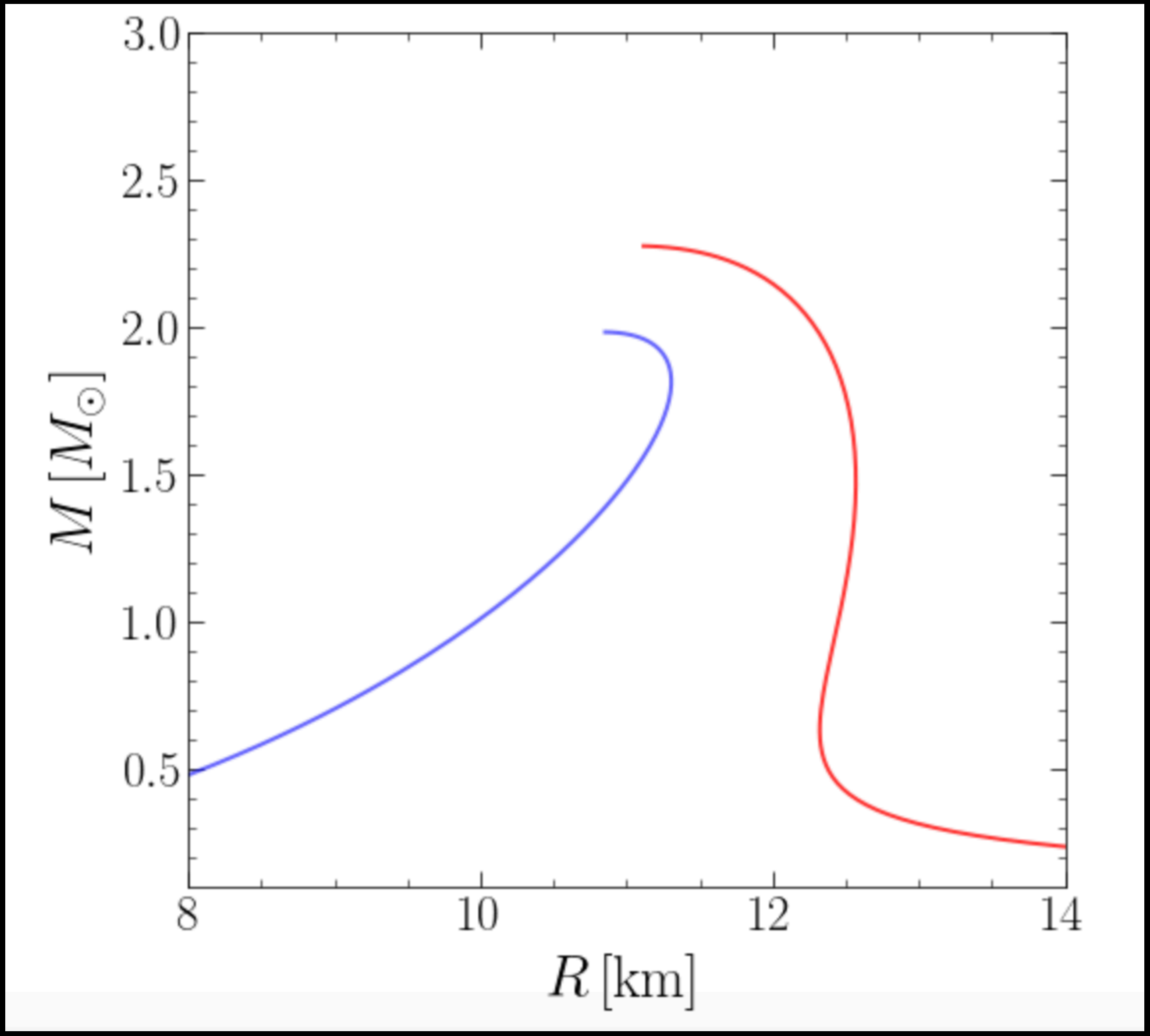
More stiff, larger maximal mass





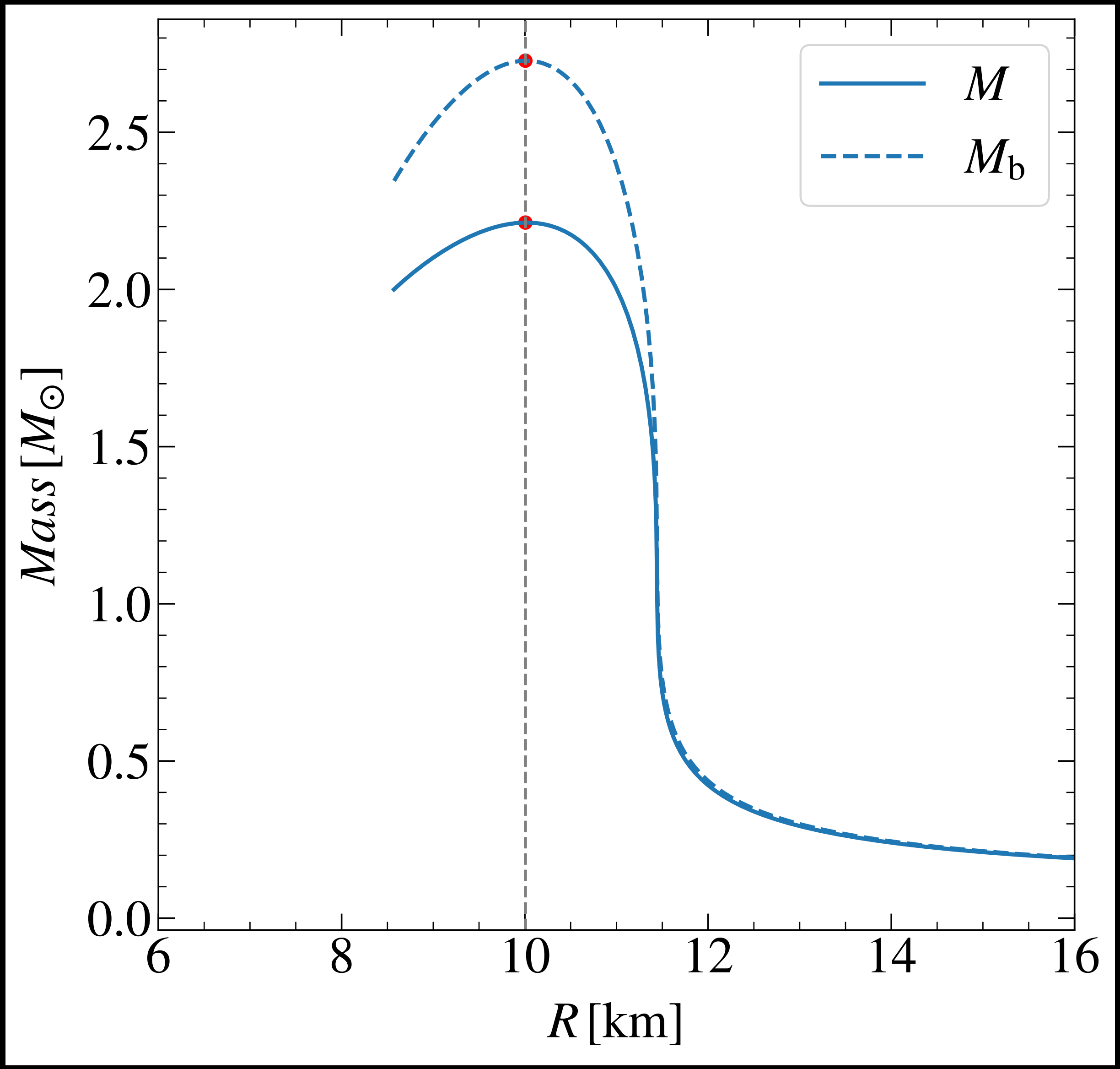
Self bound or gravitational bound?

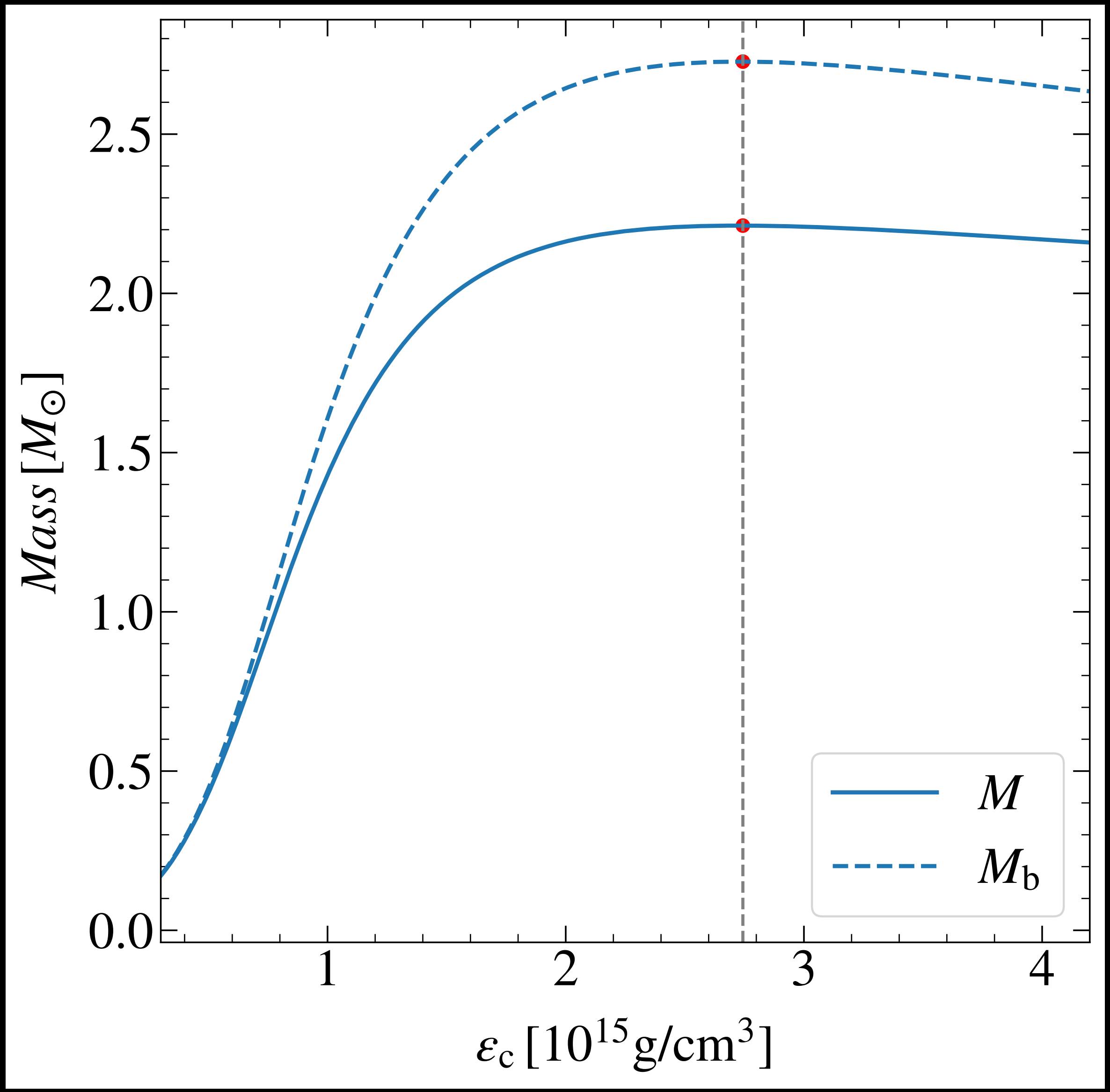




Gravitational mass and baryonic mass

$$M = m(R) = \int_0^R 4\pi r^2 \varepsilon \, \mathrm{d}r \qquad M_\mathrm{b} = 4\pi \int_0^R \frac{\rho r^2}{\sqrt{1 - 2m/r}} \, \mathrm{d}r$$

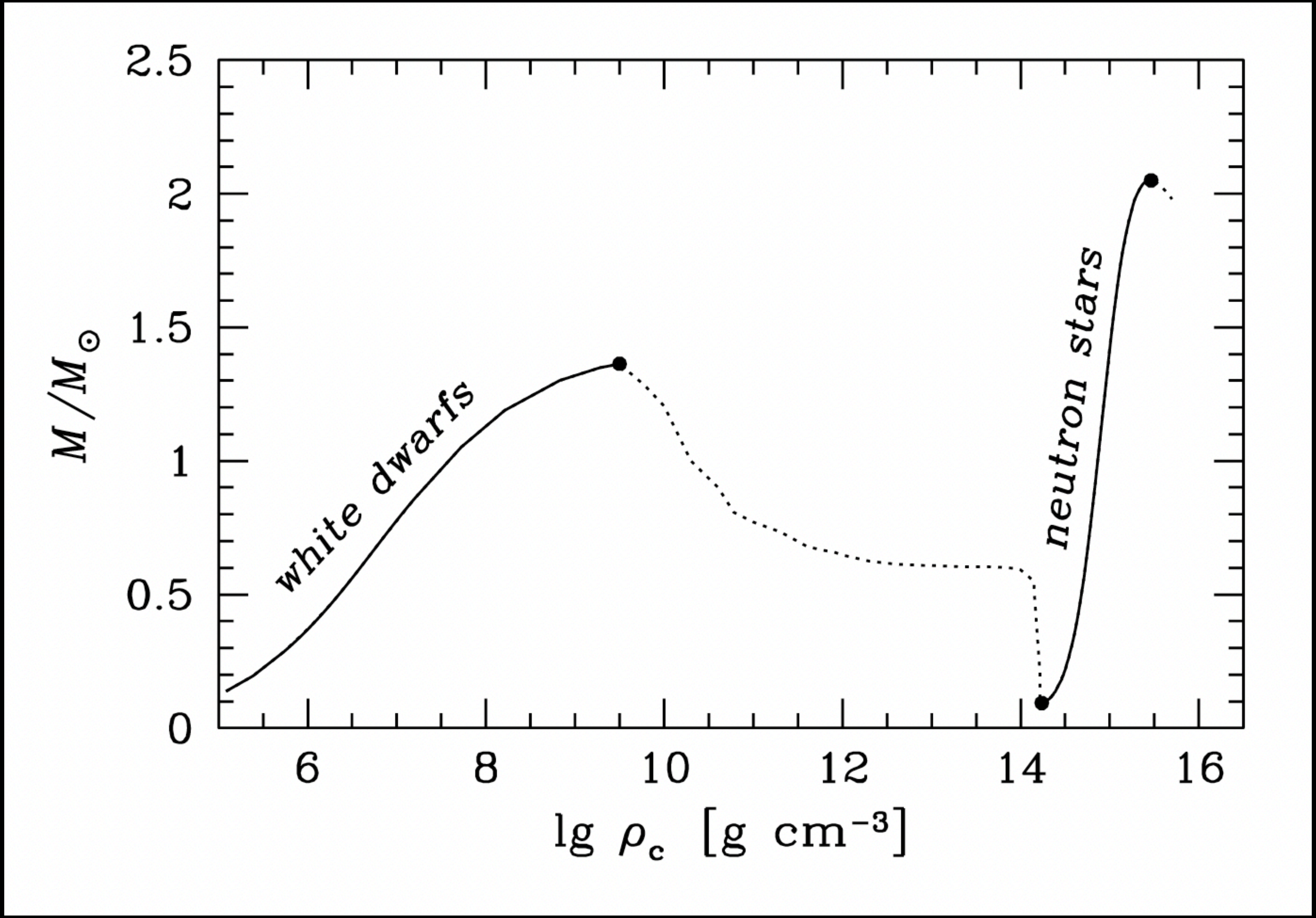




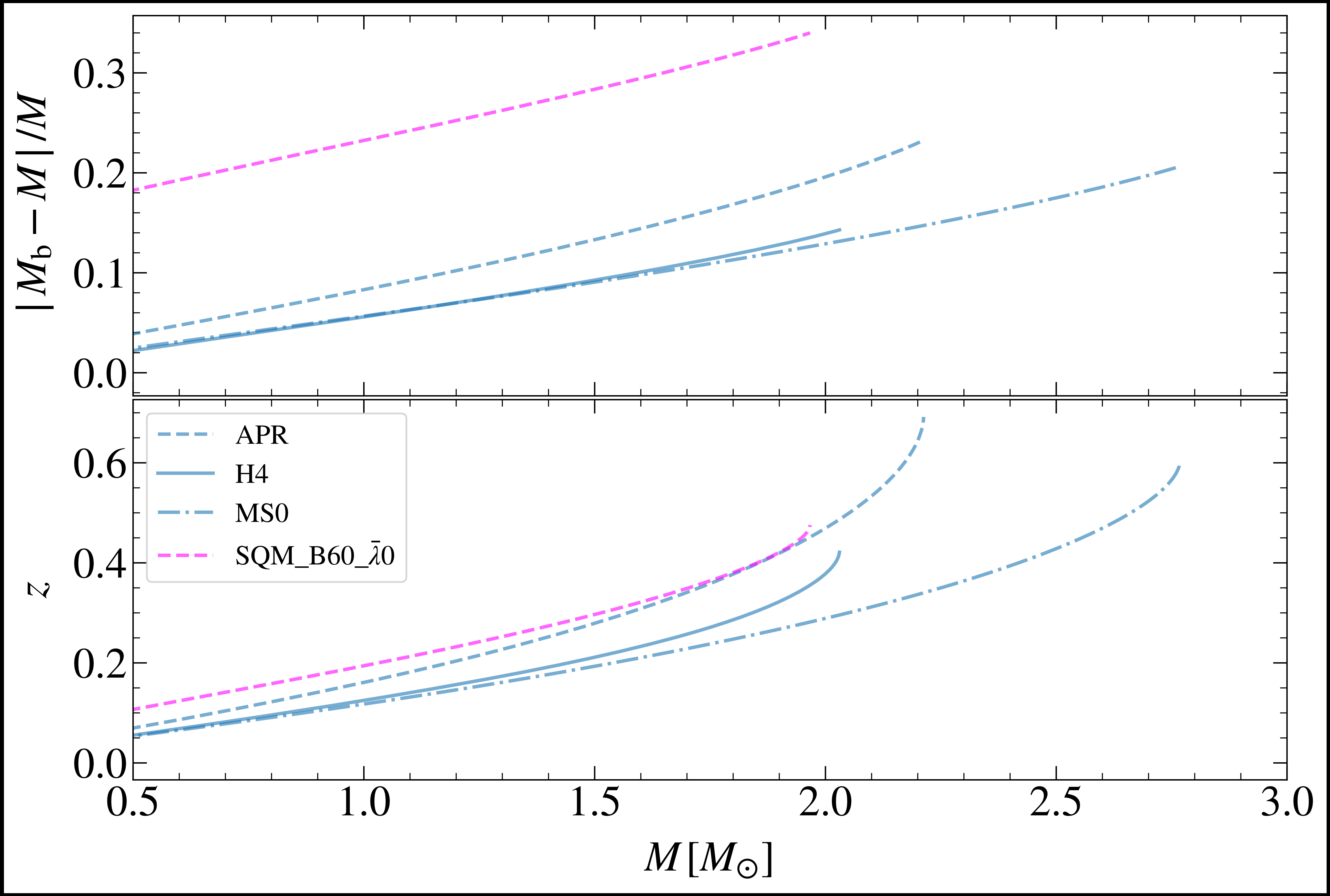
Stability of compact objects

Do we have objects between NSs and WDs?

And what happened for star beyond the maximal mass?



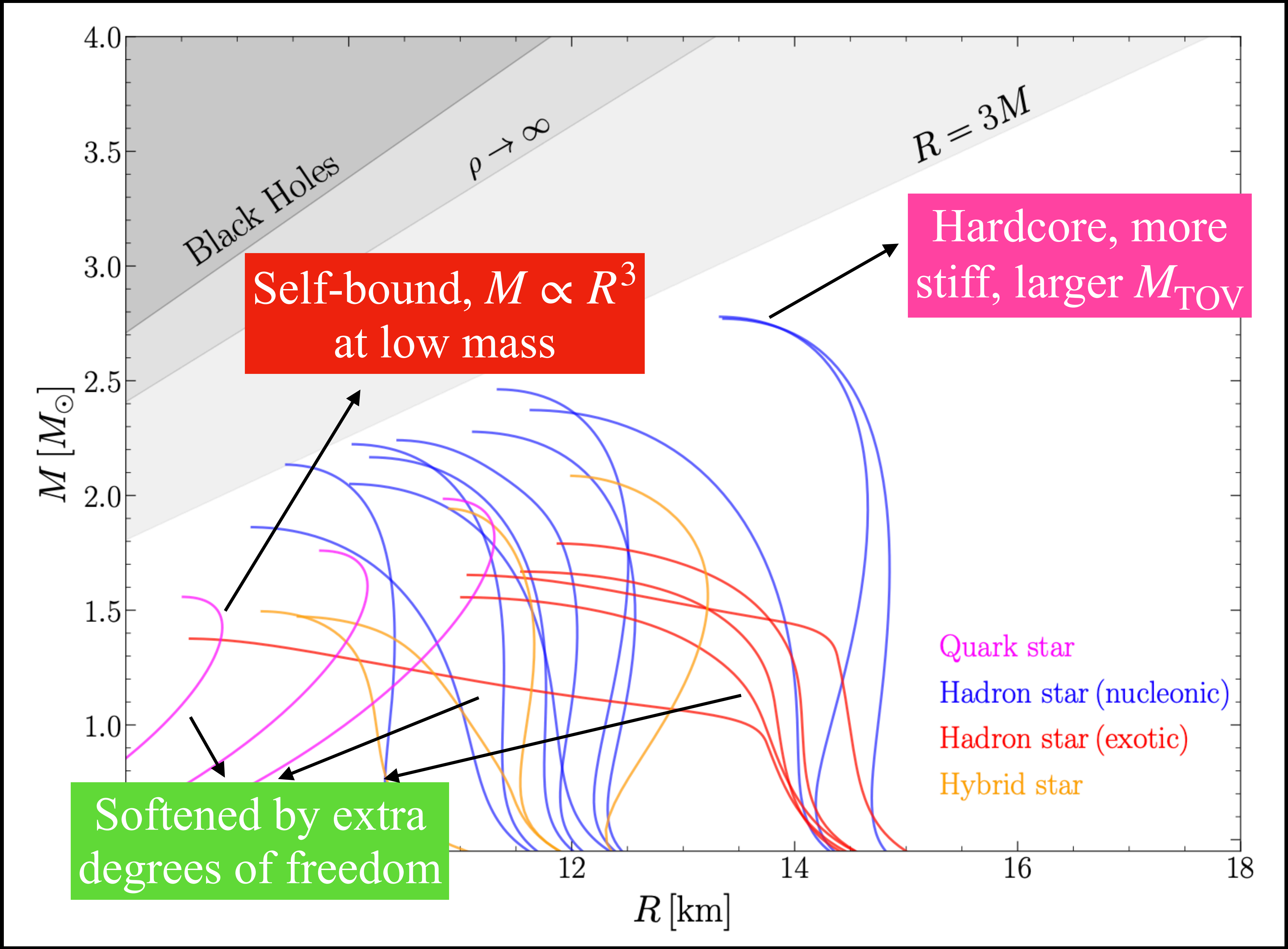
Binding energy and surface redshift



Mass-radius relation for neutron stars

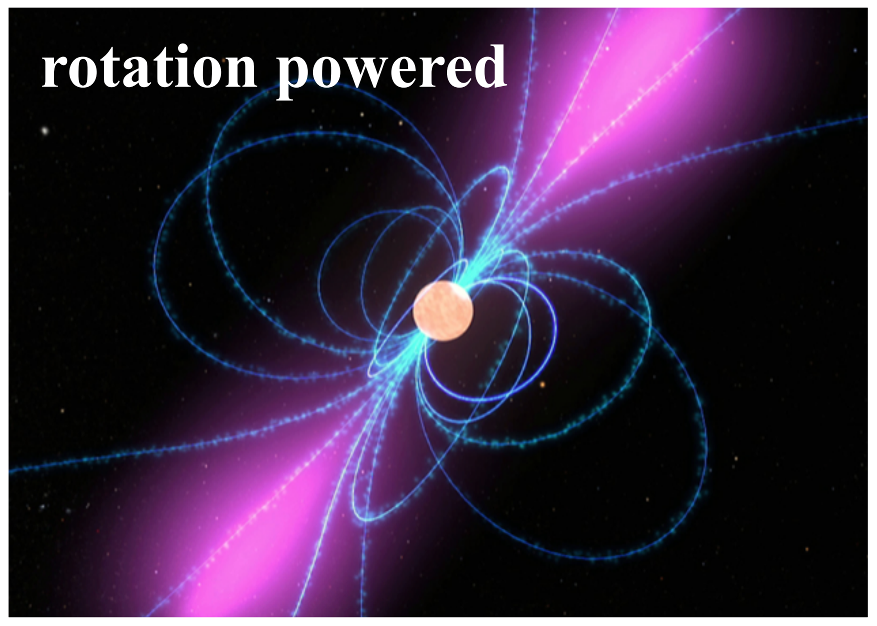
Soft (lower M_{TOV}, compact)

Stiff (higher M_{TOV}, extended)



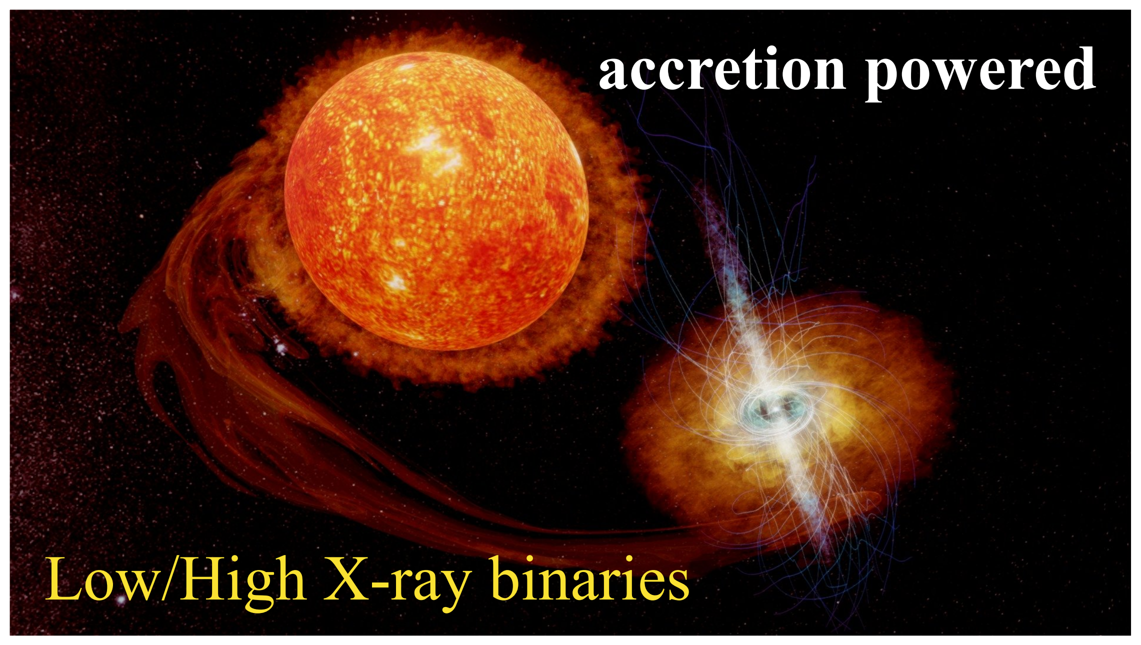
Many faces of pulsars

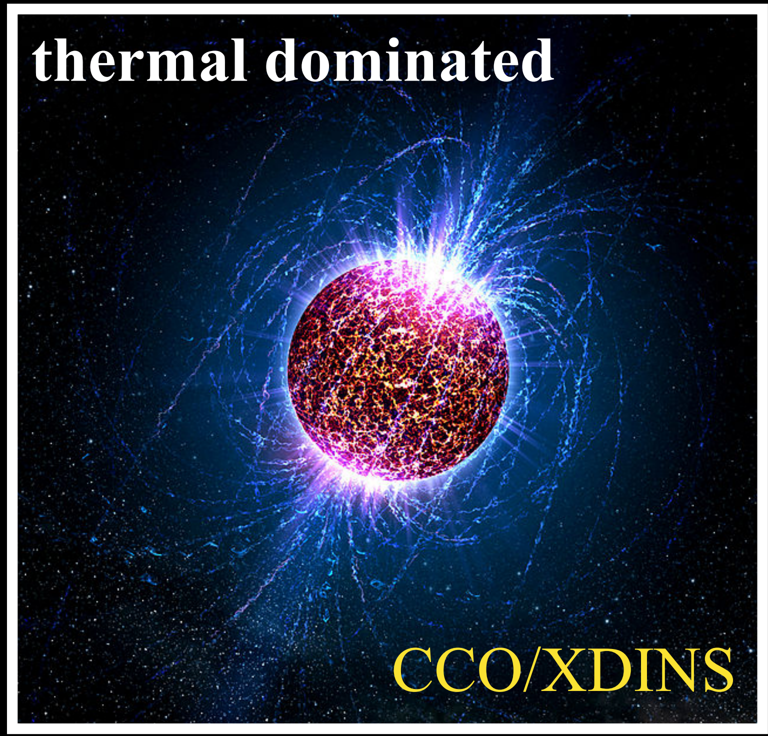
Up to now, over 3000 pulsars have been detected, according to the energy sources, they are classified as:



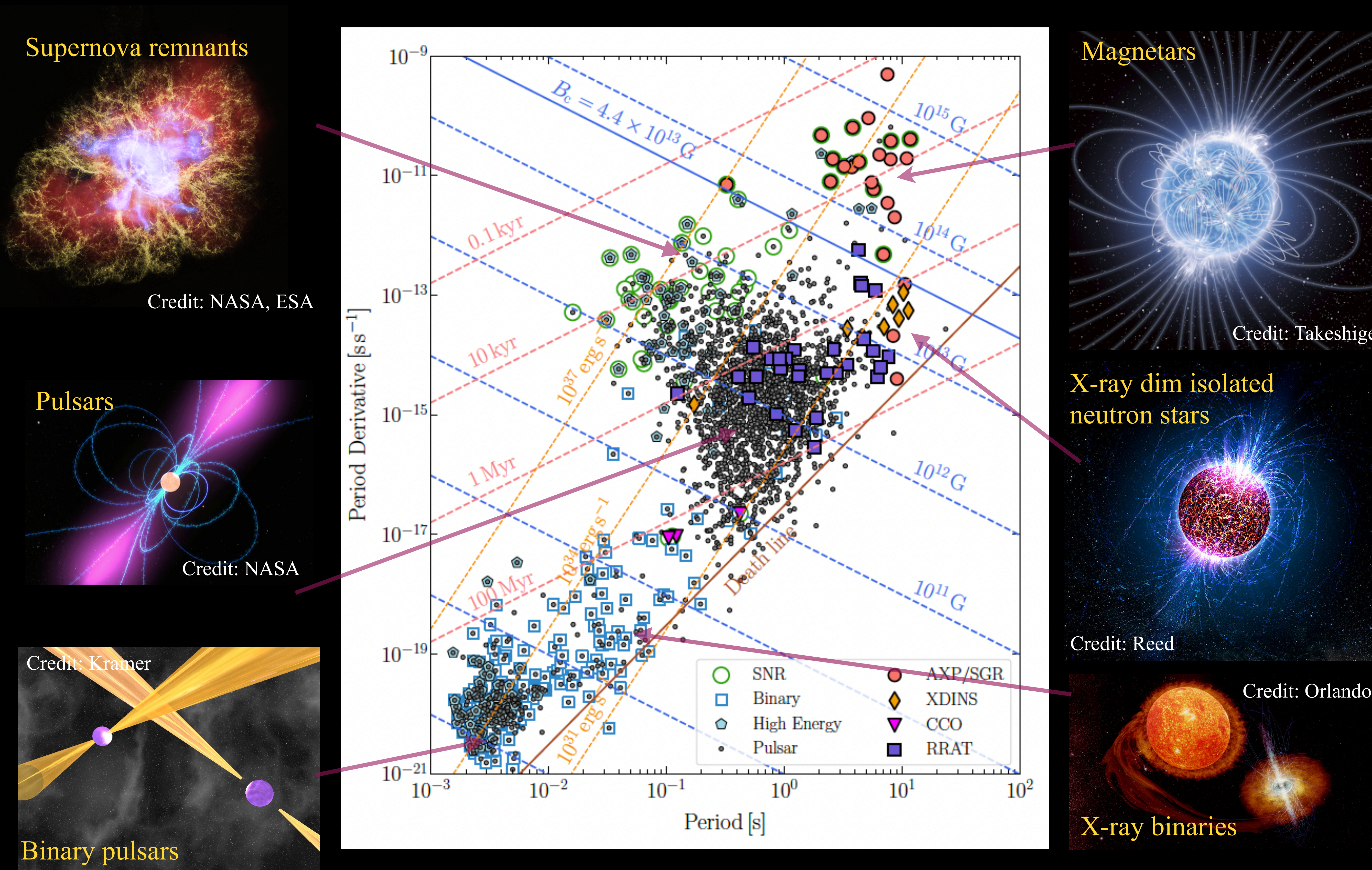
Most are isolated
10% in binaries

Companions:
ordinary stars,
white dwarfs,
neutron stars,
planets
still missing: black hole



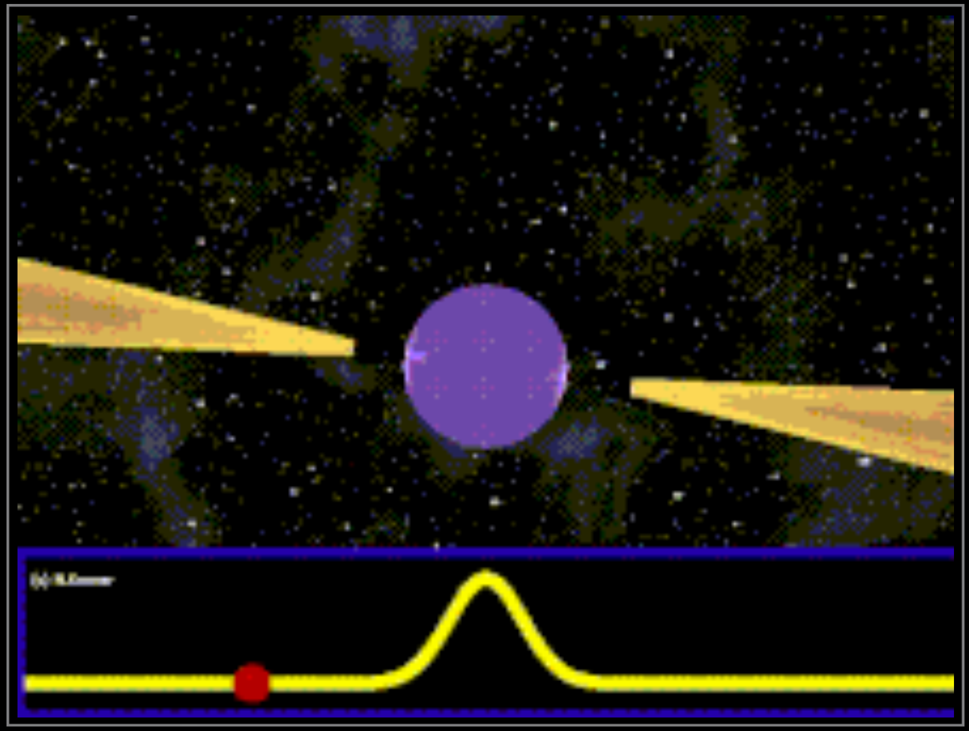


The many faces of neutron stars

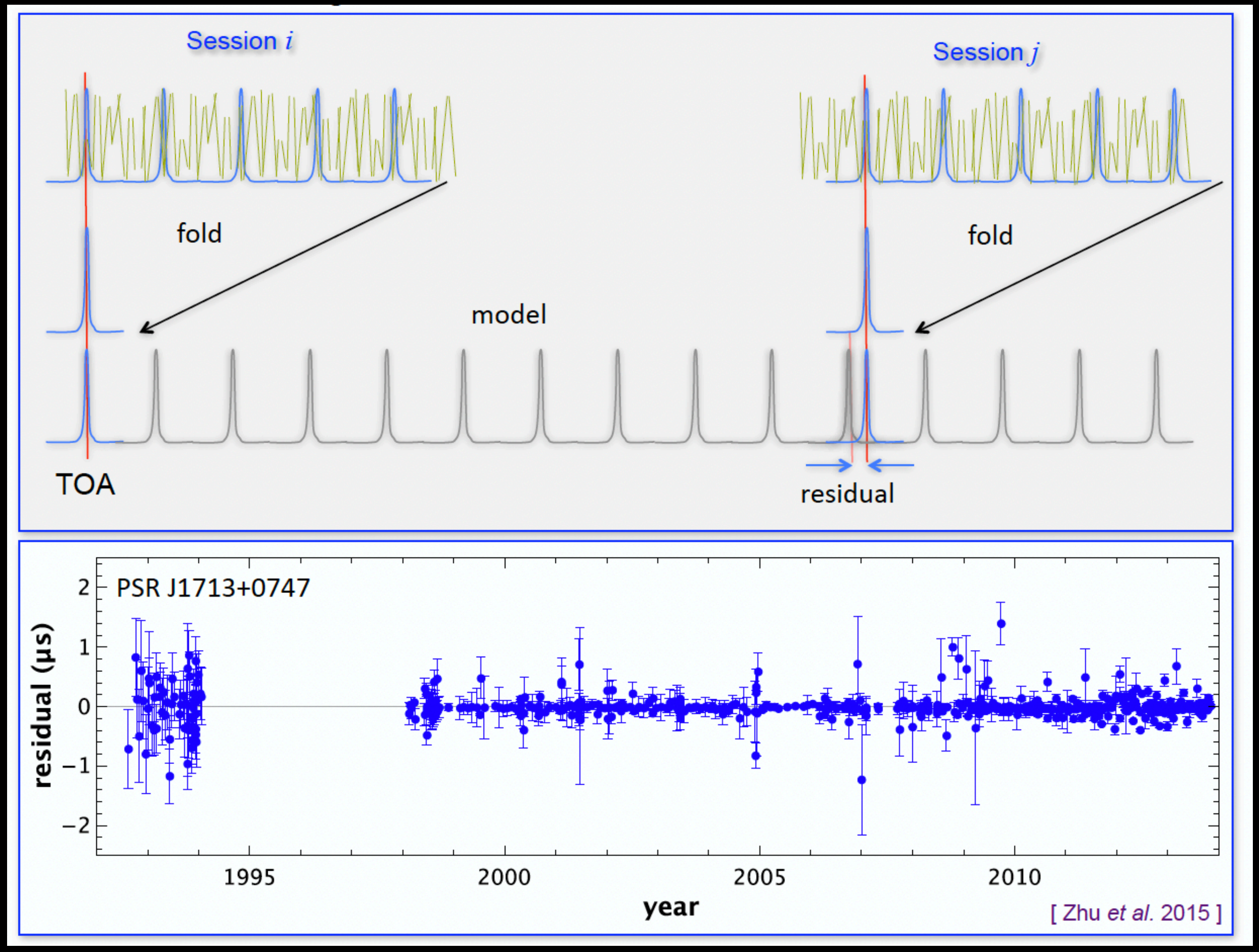


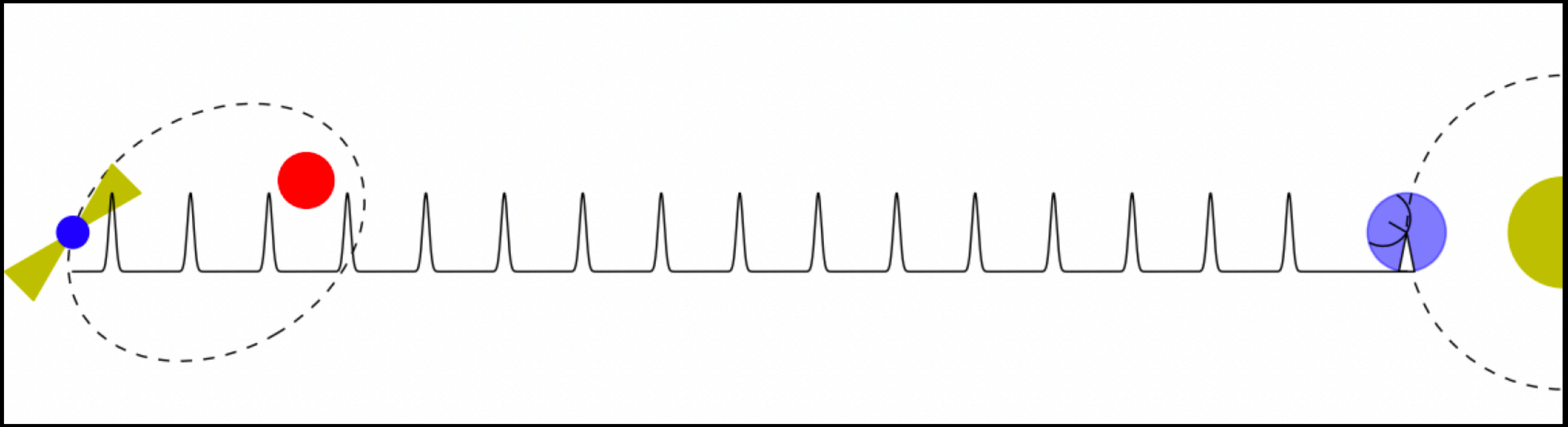
Data taken from ATNF Pulsar Catalog and McGill Catalog

Pulsars are clocks

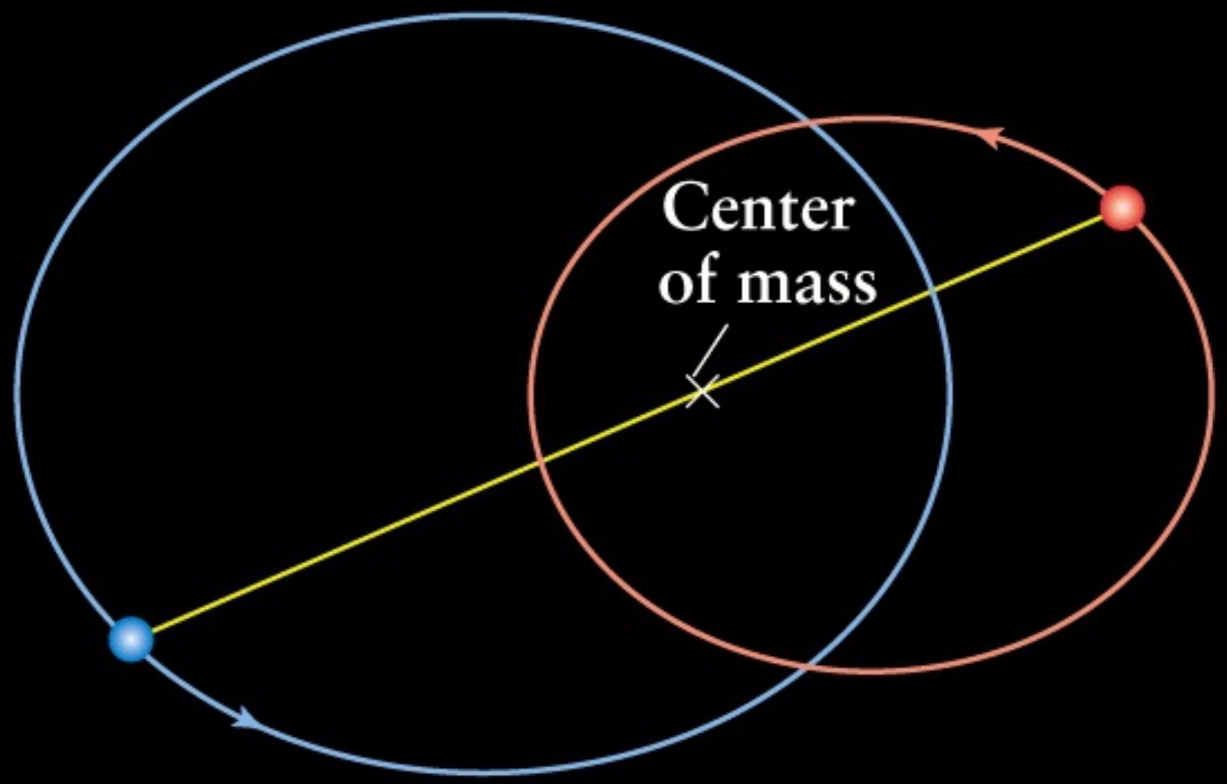


$P = 2.947108069160717(3)$ ms





Measure mass



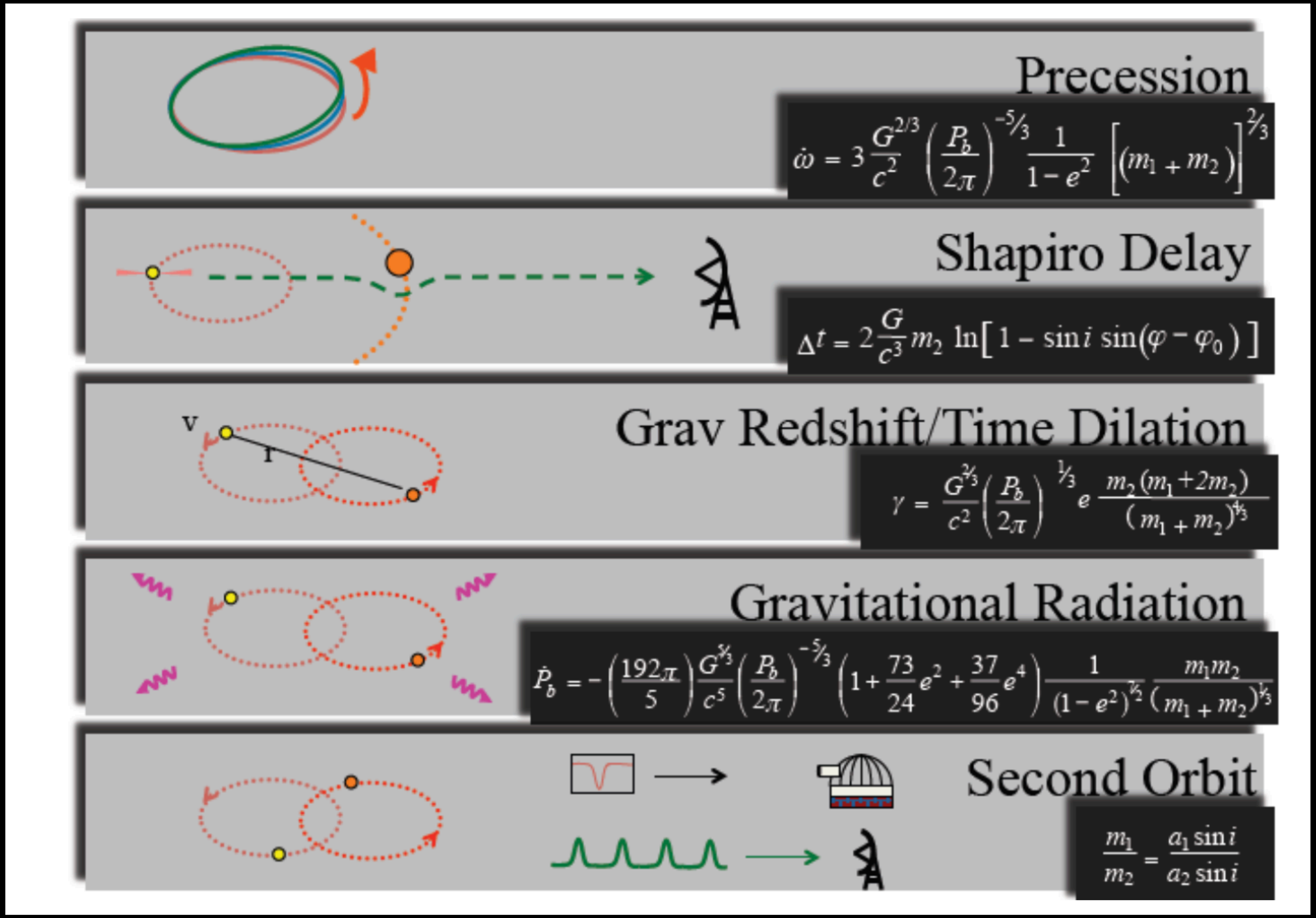
Play with the Kepler's law

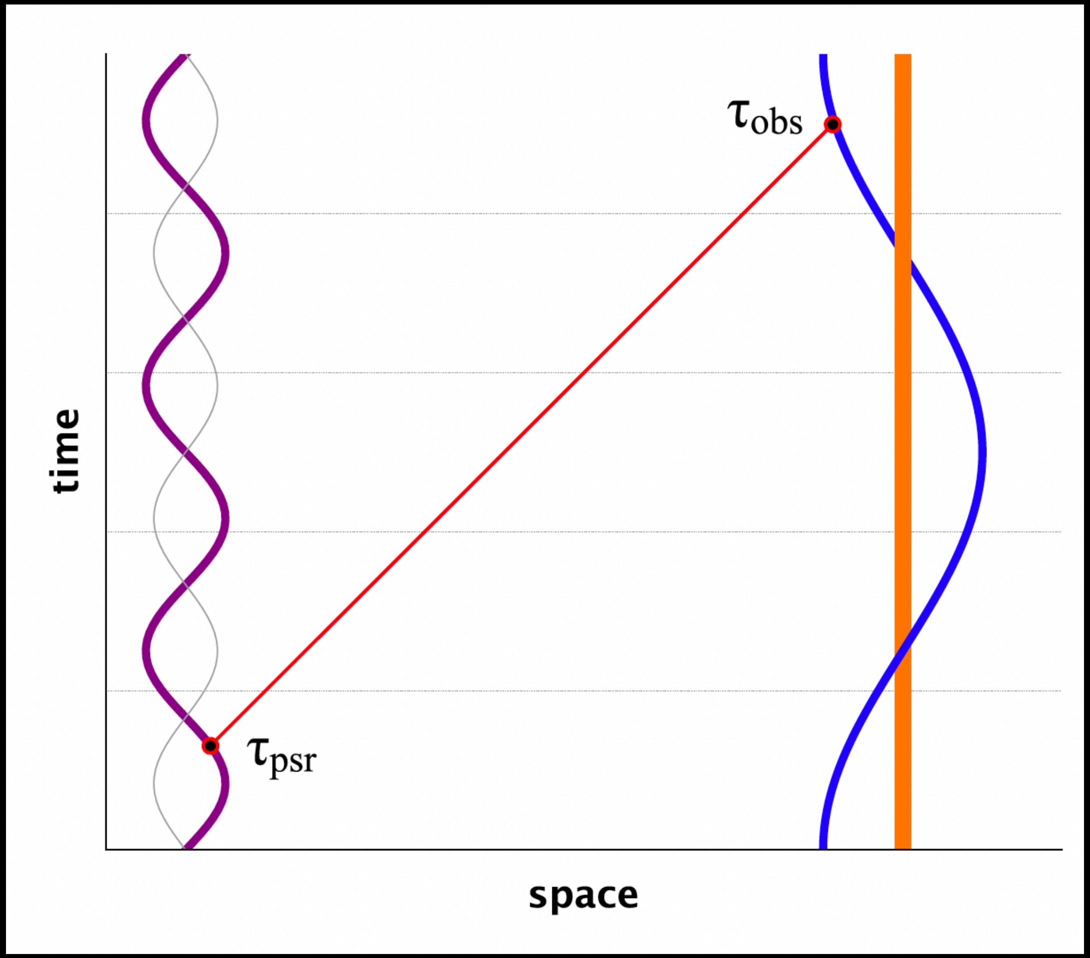
$$f = \frac{\left(M_{\mathrm{c}} \sin i\right)^3}{M_{\mathrm{T}}^2} = \frac{4\pi^2}{T_{\odot}} \frac{x_{\mathrm{psr}}^3}{P_{\mathrm{b}}^2}$$

We have three unknowns $\left(i, M_{\mathrm{psr}}, M_{\mathrm{c}}\right)$

Measure mass: relativistic orbit

Relativistic corrections

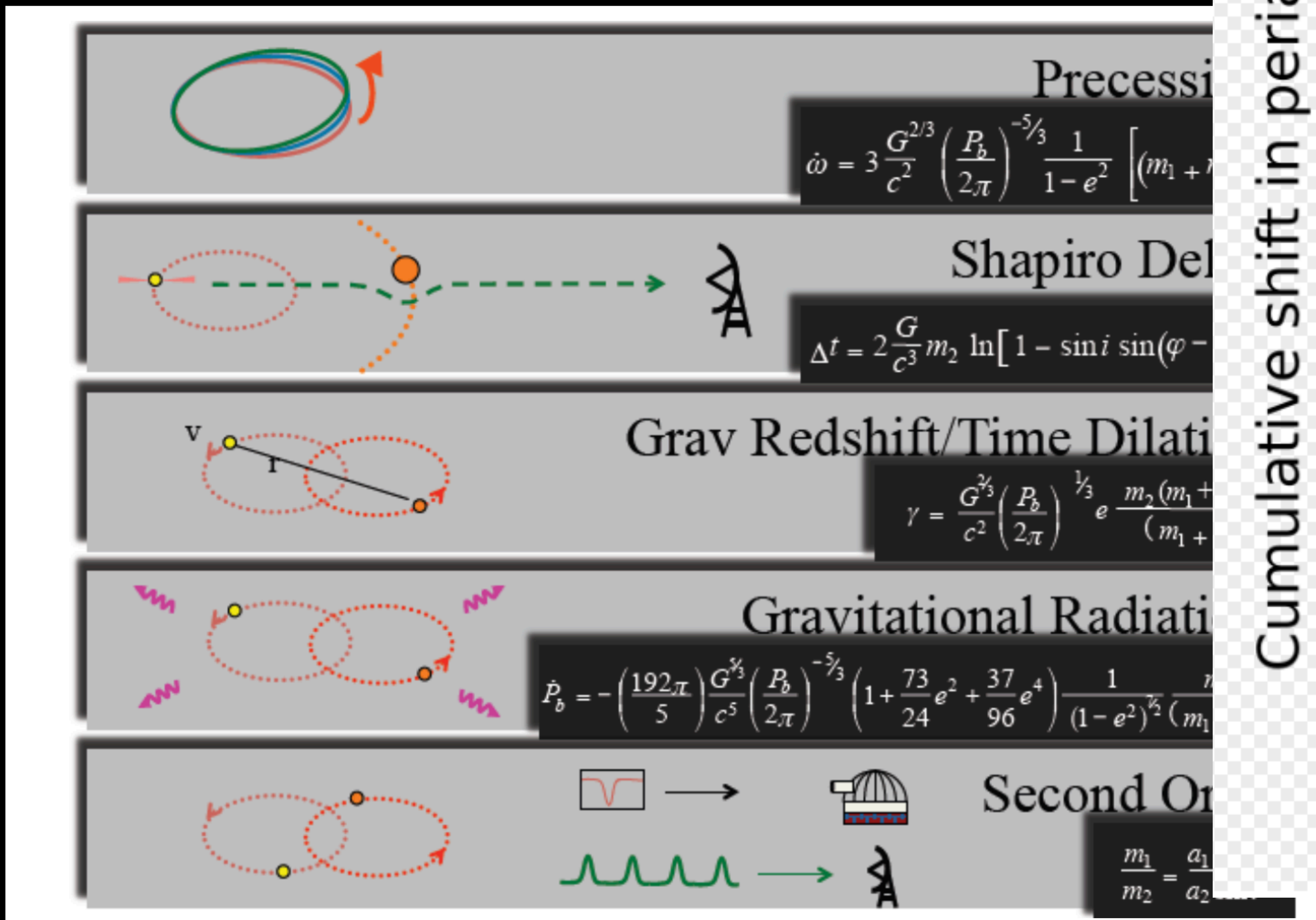


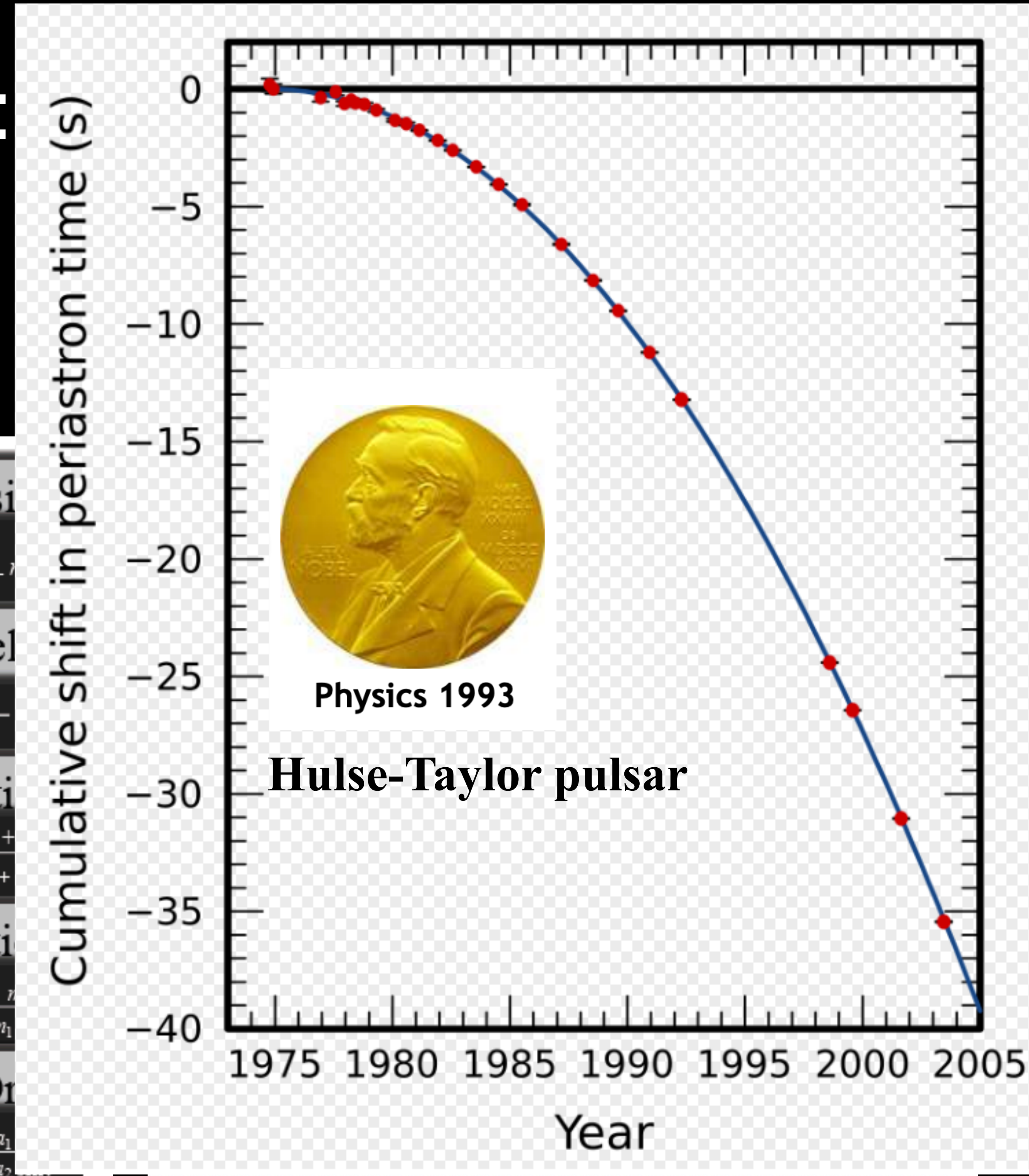


Any PK measurement yields a line in the (m_1, m_2)-plane. Hence, two PK parameters determines m_1 and m_2 uniquely.

Measure mass:

Relativistic corrections

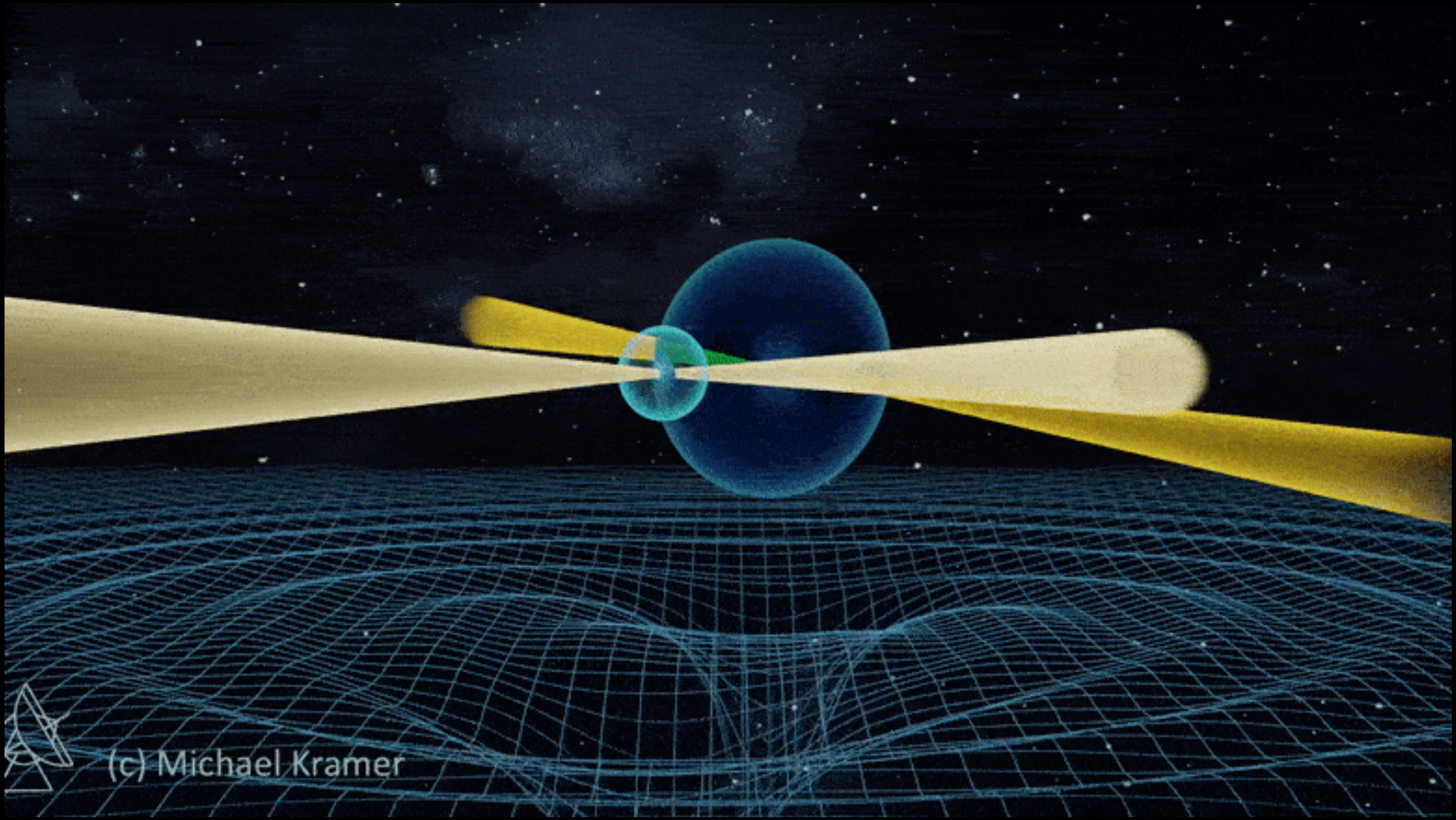




Any PK measurement yields a line in the (m_1, m_2)-plane. Hence, two PK parameters determines m_1 and m_2 uniquely.

Measure mass: relativistic orbit

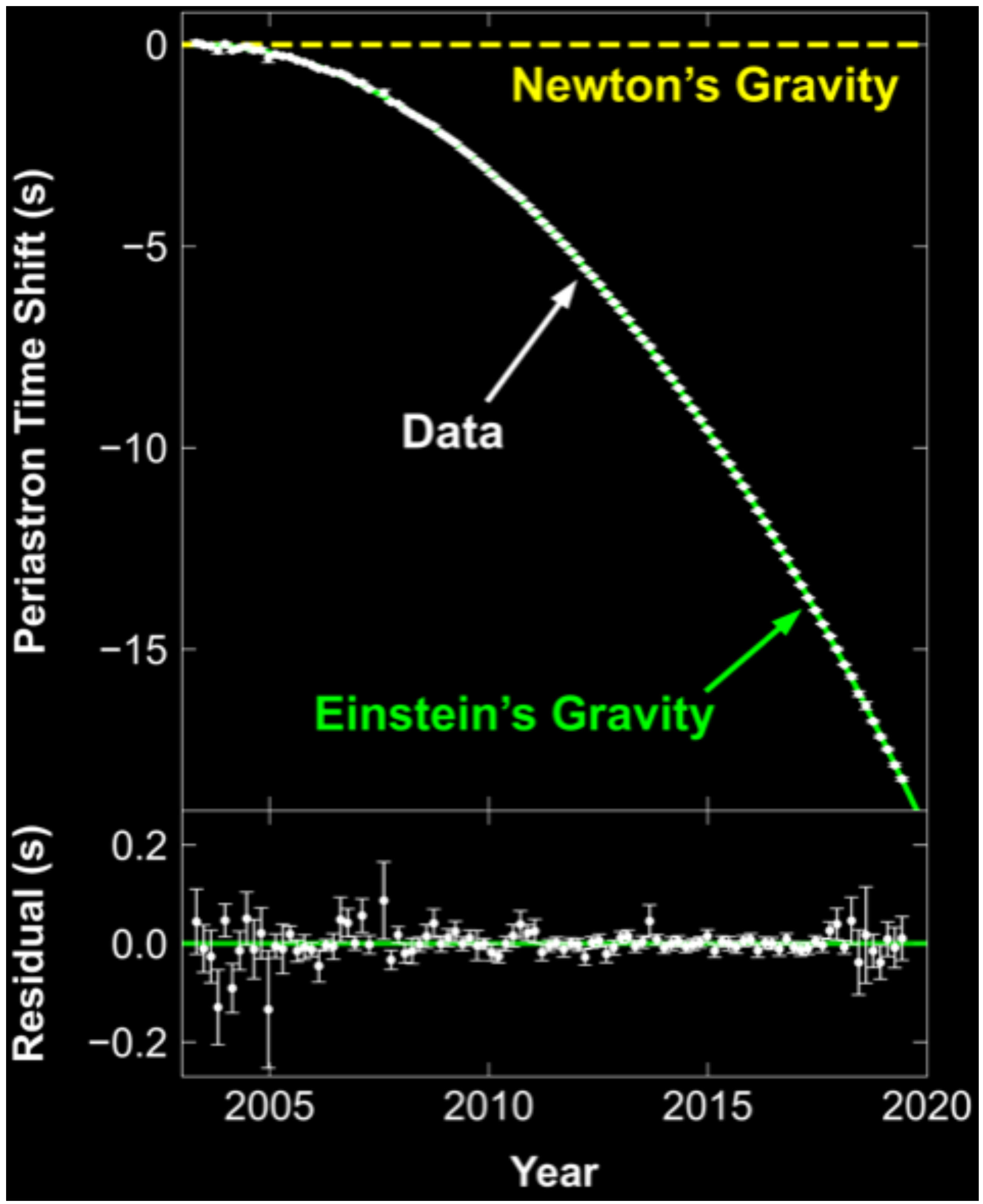
The Double Pulsar PSR J0737-3039

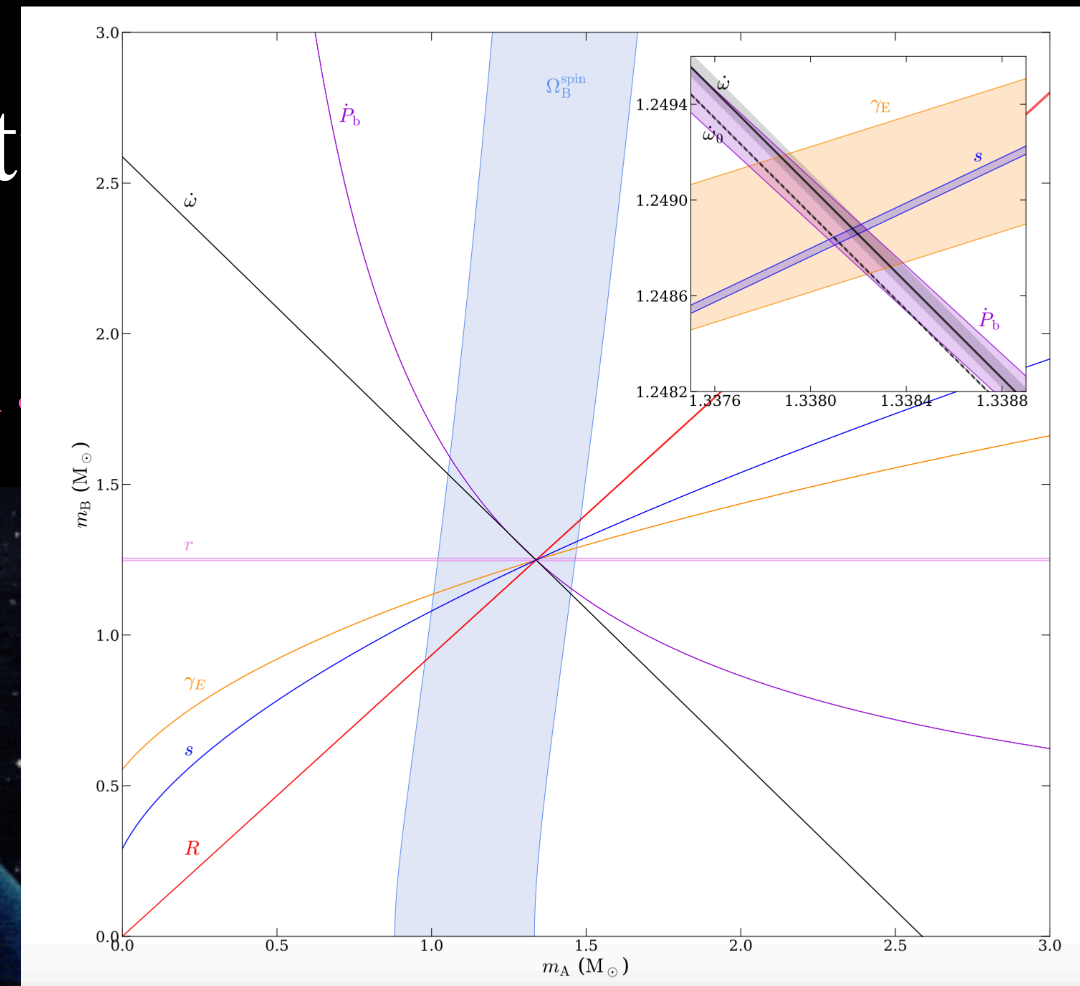


Spin period: 23ms and 2.8s   Orbit period: 2.45 hour   Inclination angle: 88.69°

relat

PSR



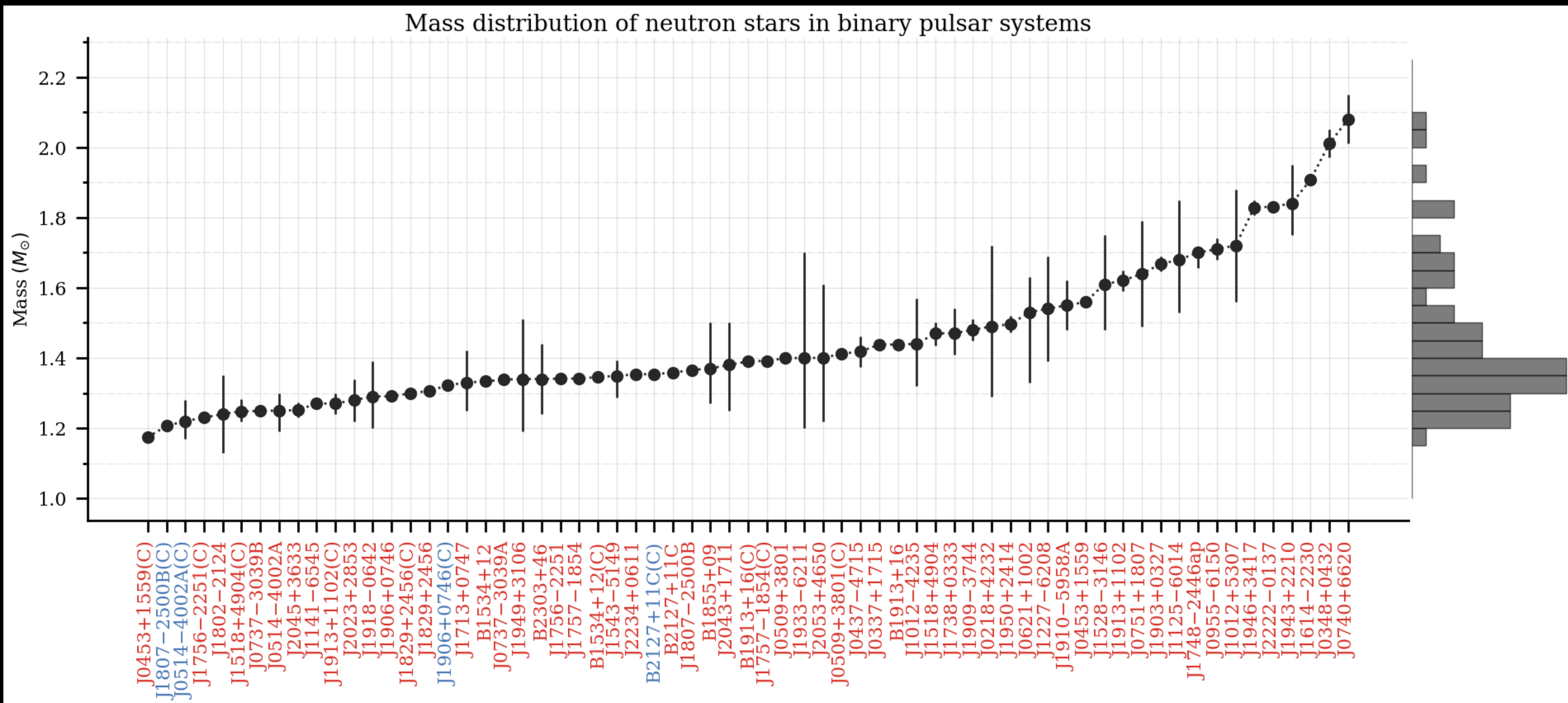


Spin period: 23ms and 2.8s

Orbit period: 2.45 hour

Inclination angle: 88.69°

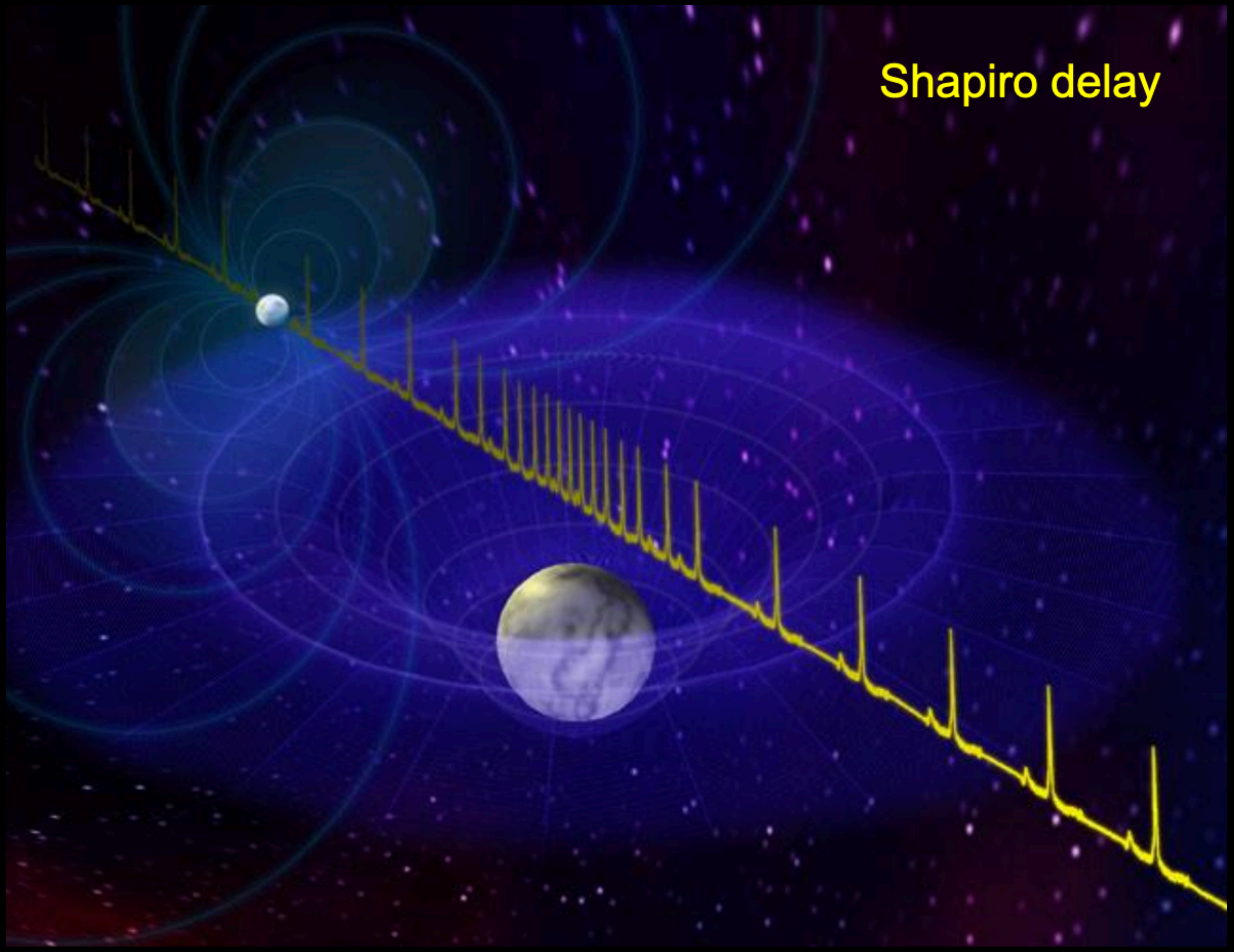
Mass measurement



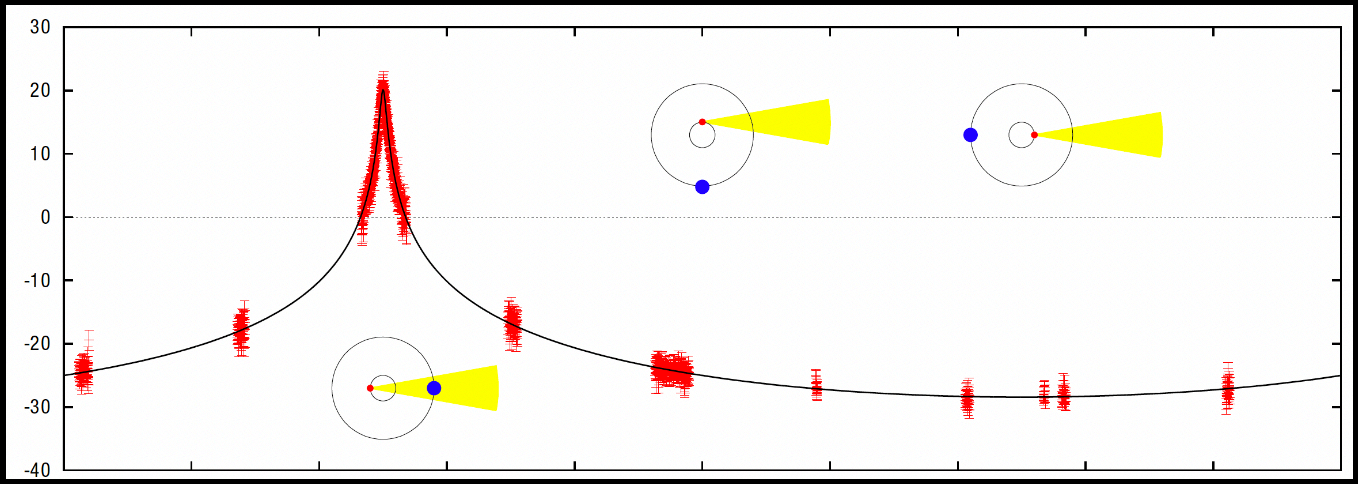
Measure mass

Massive pulsars can set lower limit of maximum mass—> constrain EoS

For the vast majority of pulsar-WD systems, the only measurable PK parameters are those related to the Shapiro delay.

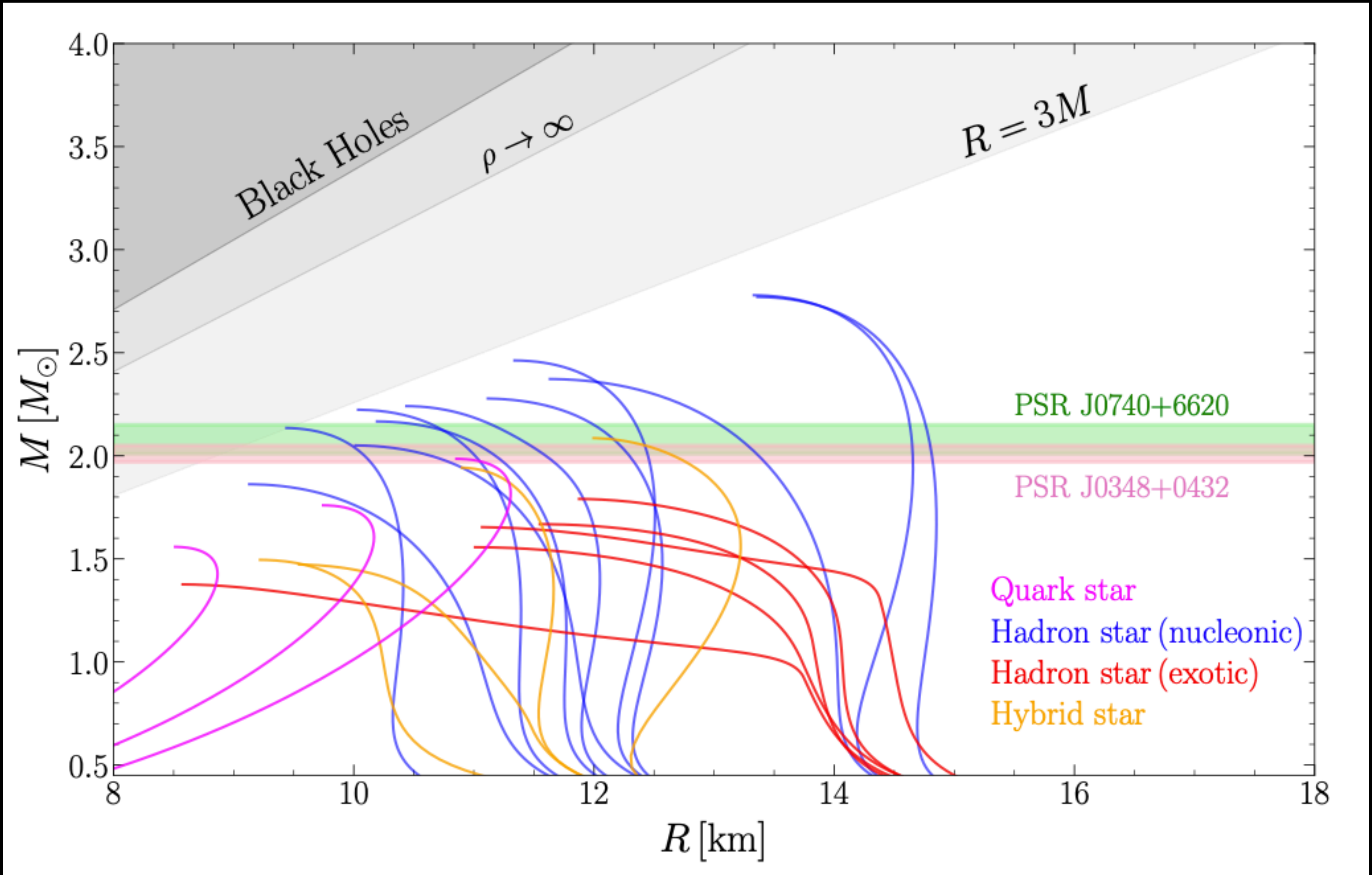


Demorest et al., 2010



$$M_p = 1.97(4)\,M_\odot$$
$$M_c = 0.500(6)\,M_\odot$$

Measure mass



Where is the limit?

Maximal mass set by causality

Incompressible fluid

$$M_{\max}^{\rm inc} = \frac{4\pi}{9}\frac{R_{\max}c^2}{G} = \frac{4c^3}{3^{5/2}\pi^{1/2}G^{3/2}\rho_{\rm inc}^{1/2}} \approx 5.09 M_\odot \left(\frac{5\times10^{14}\ {\rm g\ cm^{-3}}}{\rho_{\rm inc}}\right)^{\frac{1}{2}}$$

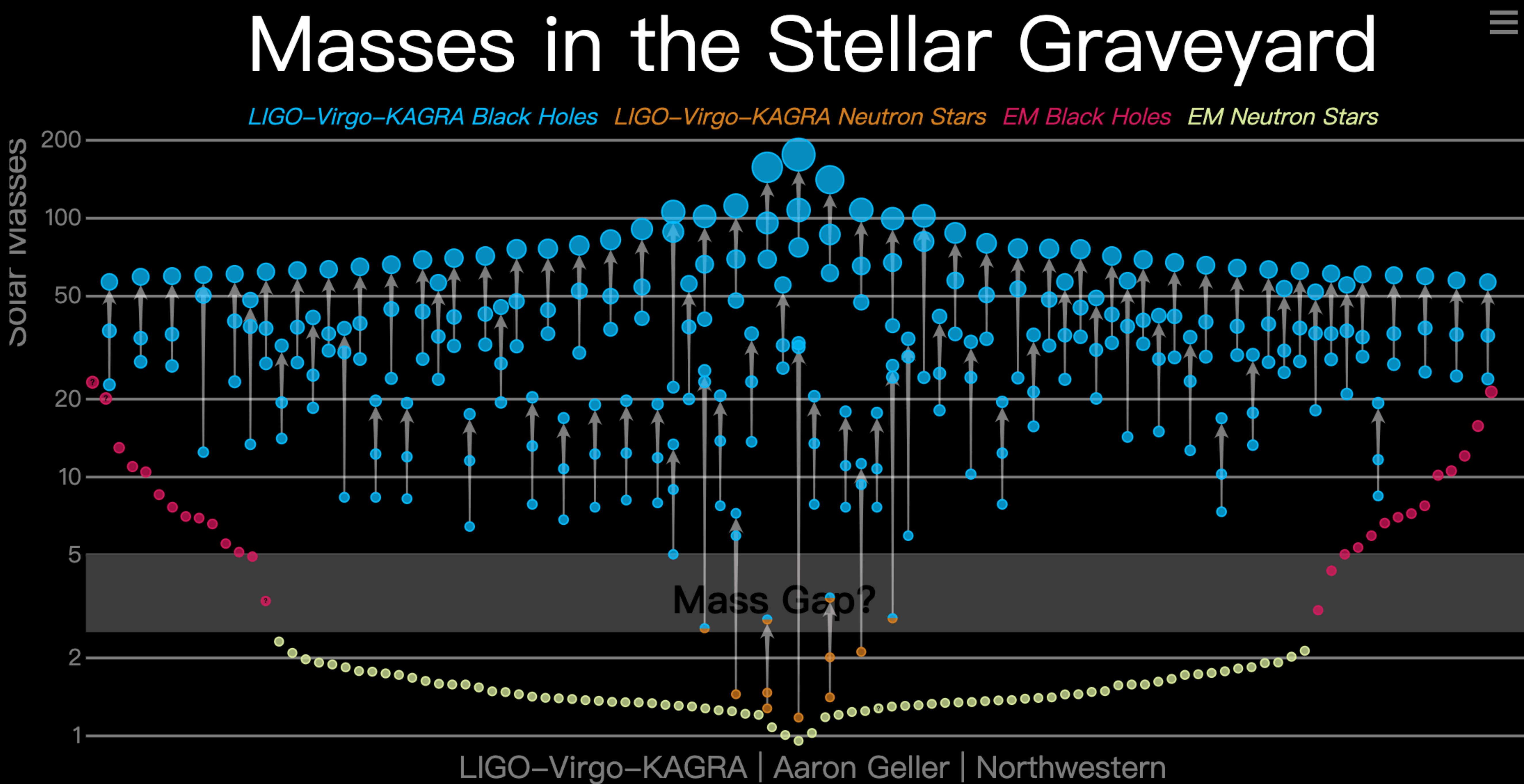
Causality limit

$$P_{>}^{\rm CL}(\rho) = P_{\rm u} + \left(\rho - \rho_{\rm u}\right)c^2$$

General Relativity
and $v_s \leq$ c

$$\Longrightarrow M_{\max} \leq M_{\max}^{\rm CL} = 3.0\sqrt{\frac{5\times10^{14}\ {\rm g\ cm^{-3}}}{\rho_{\rm u}}}M_\odot$$

It seems safe to state that the actual $M_{\max}$ of neutron stars built of baryonic matter is below 3 $M_\odot$

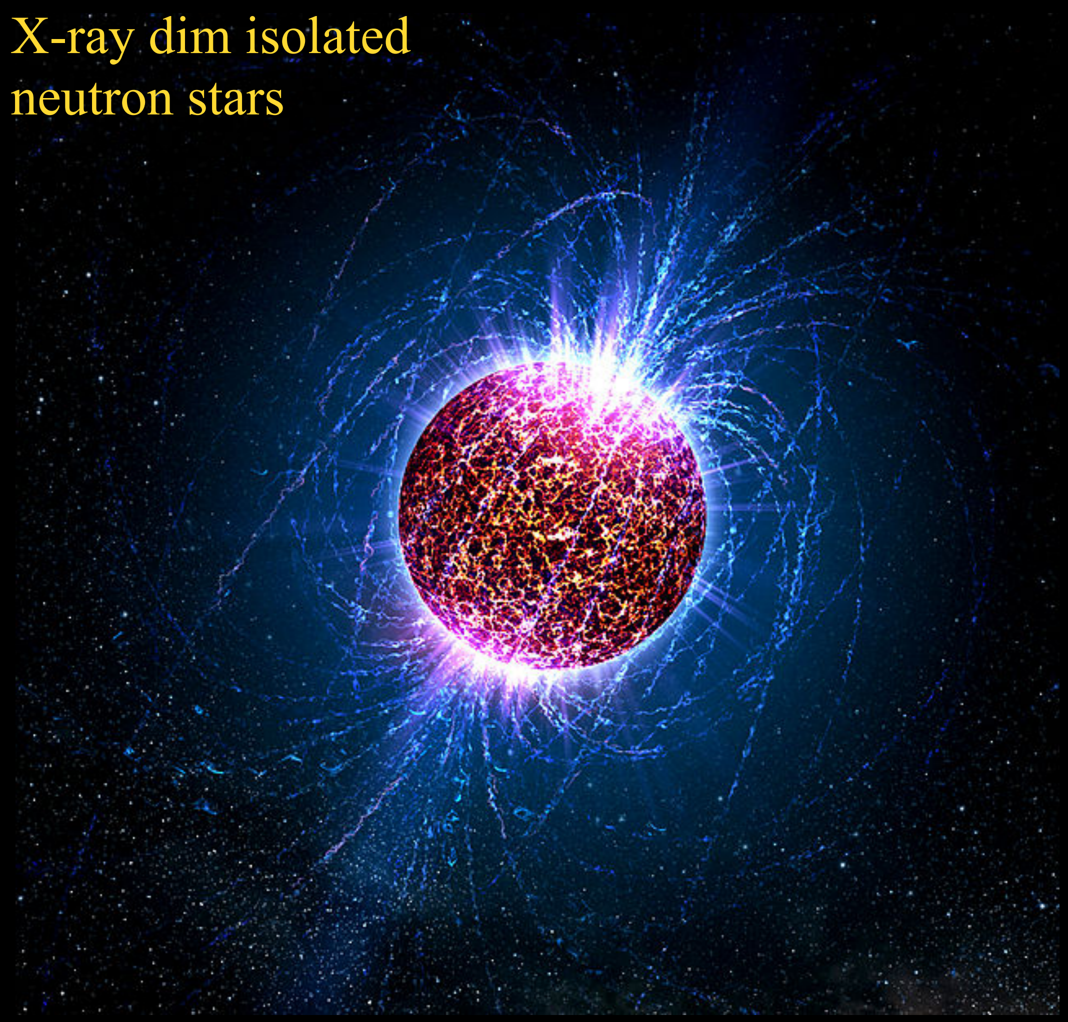


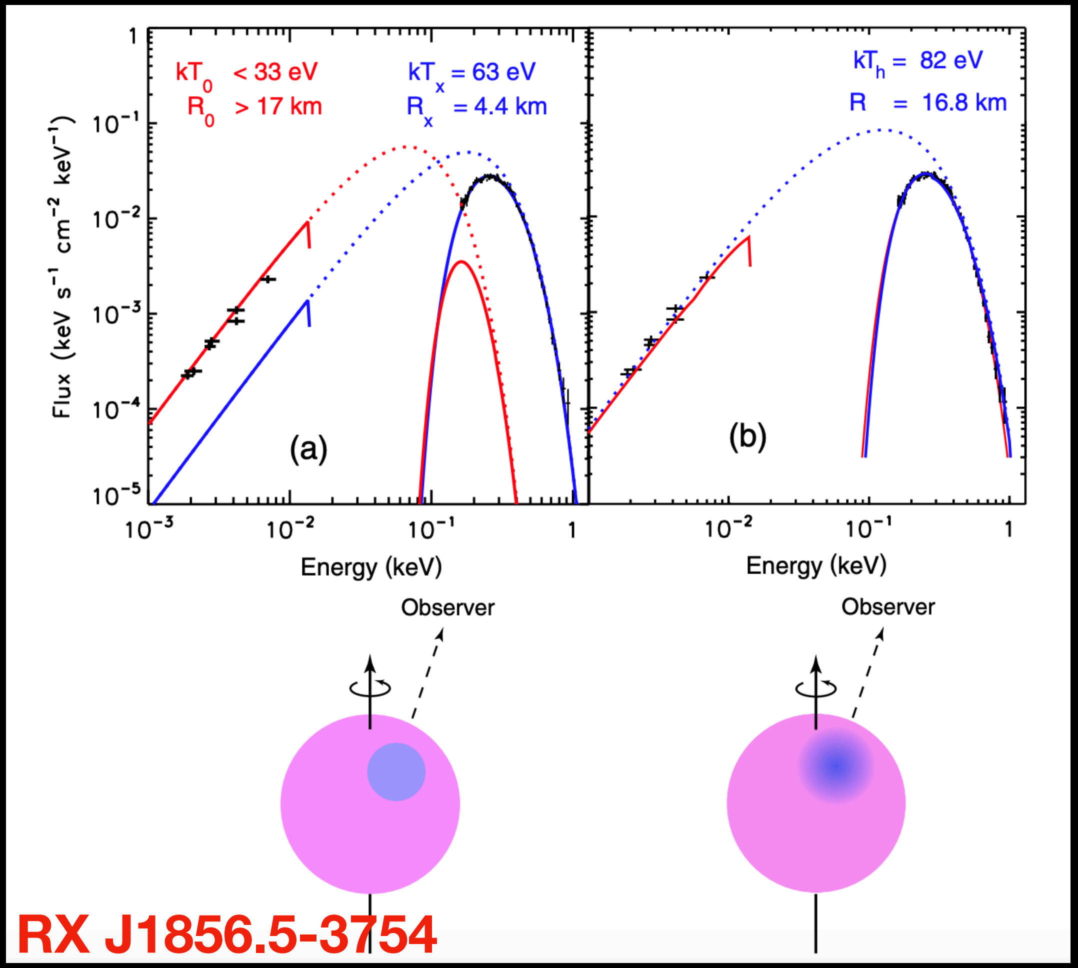
Where is the limit?

Measure radius—it's hard

- Measure surface thermal emission; Quiescent LMXBs, X-ray Bursts, XDINS (Chandra, XMM, Athena)

$$\frac{R_{\rm obs}}{D} = \left(\frac{F_{\rm bol}}{\sigma_B T_{\rm eff}^4}\right)^{1/2} \qquad R_{\rm obs} = \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} R$$





In practice: spectral hardening (atmospheric corrections), and uncertainties in distance estimates.

Measure radius—X-ray timing

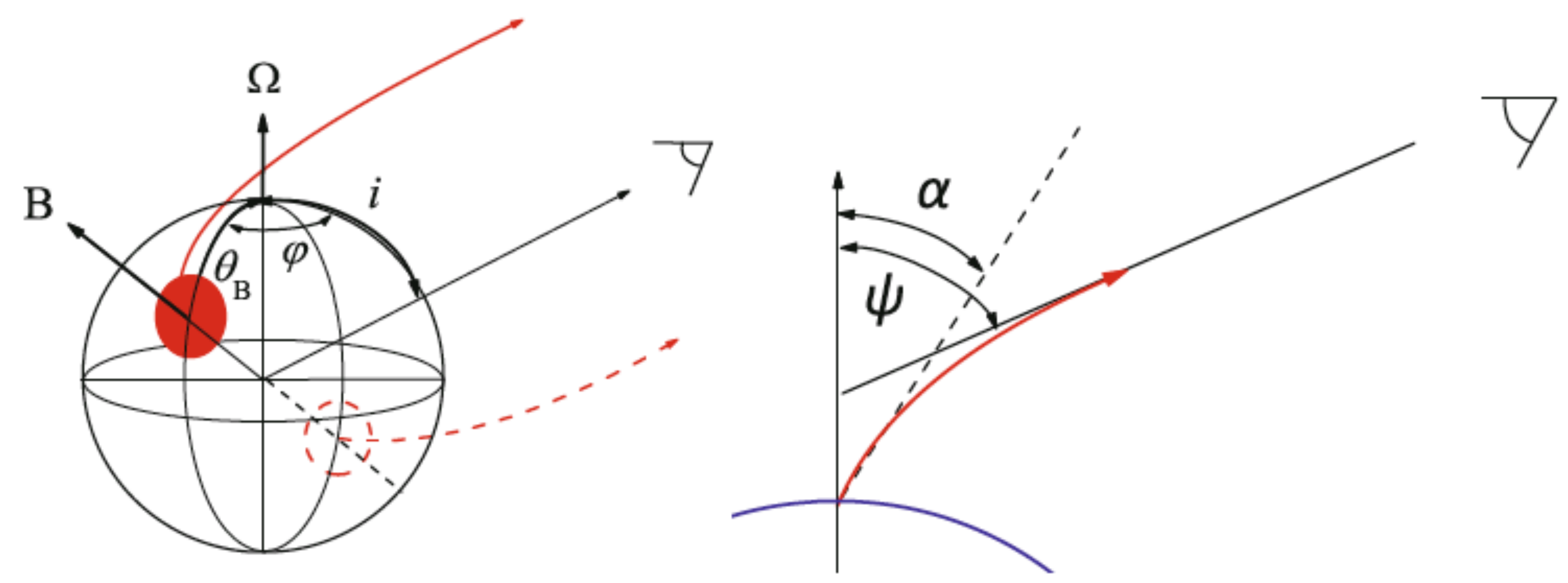


Fig. 5.11 Left: Geometry of a hotspot on the neutron star surface and relevant angles. B and Ω indicate the magnetic and rotational axis, respectively, i is the inclination angle of the rotation axis to the line of sight, θ_B is the hotspot co-latitude, and φ is the rotational phase angle. Right: The emitted α and the observed ψ angles of a light ray from the neutron star normal

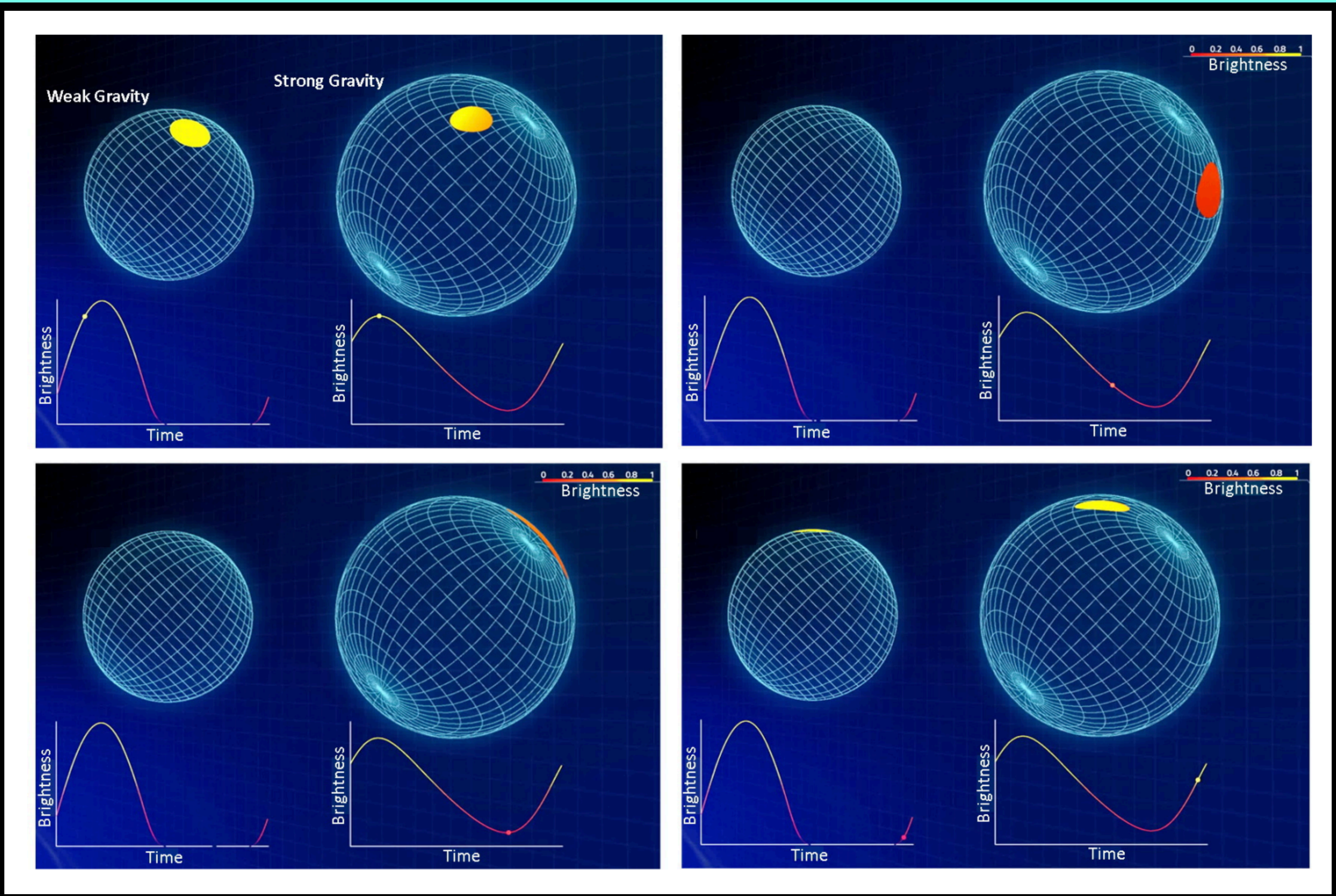
Gravitational light bending: $\psi = \int_R^\infty \frac{dr}{r^2}\left[\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{R_S}{r}\right)\right]^{-1/2}$ $\quad b = R(1+z)\sin\alpha$

Doppler boost: $\delta = \frac{1}{\gamma(1-\beta\cos\xi)}$

Homework

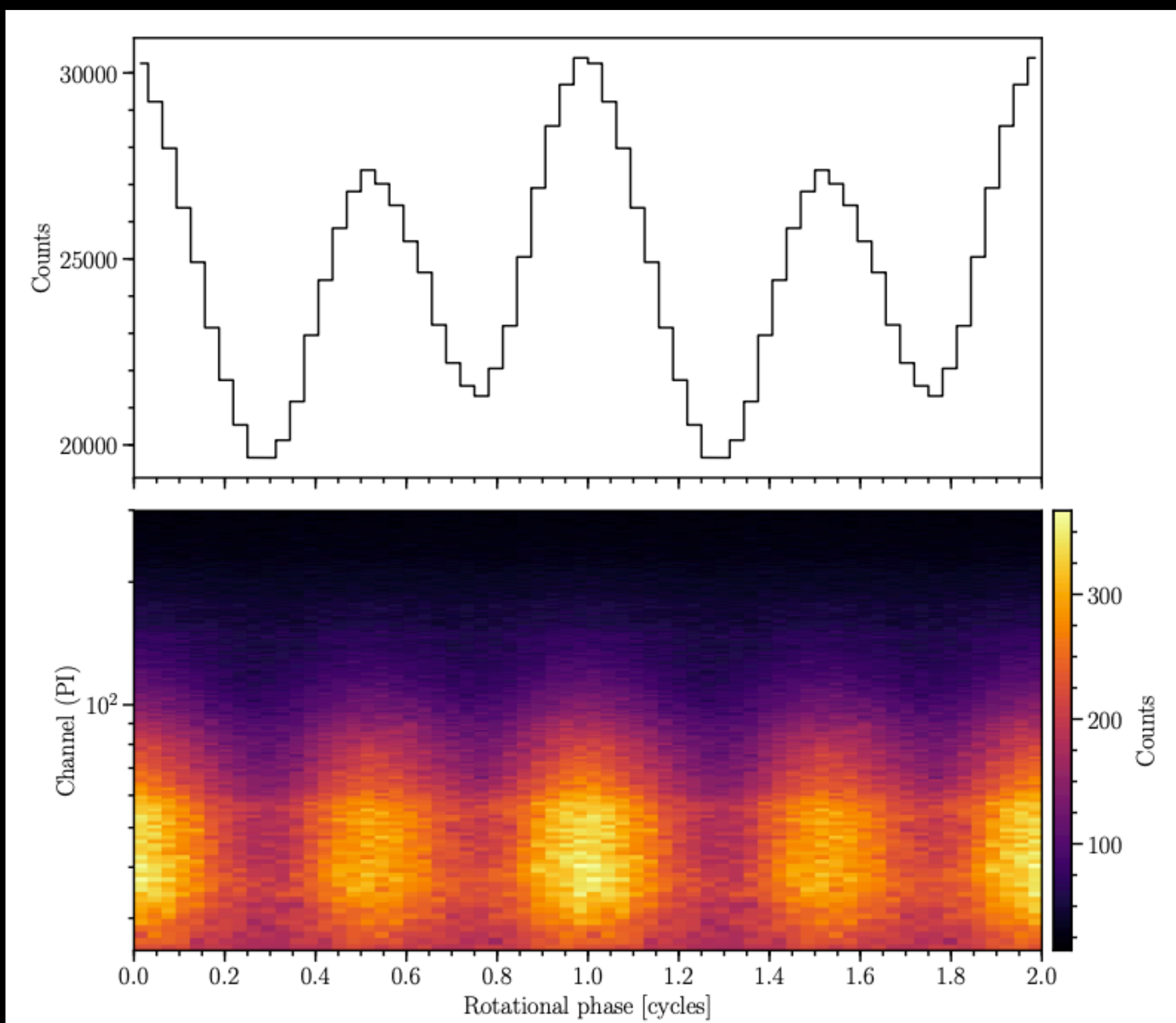
Measure radius from X-ray timing

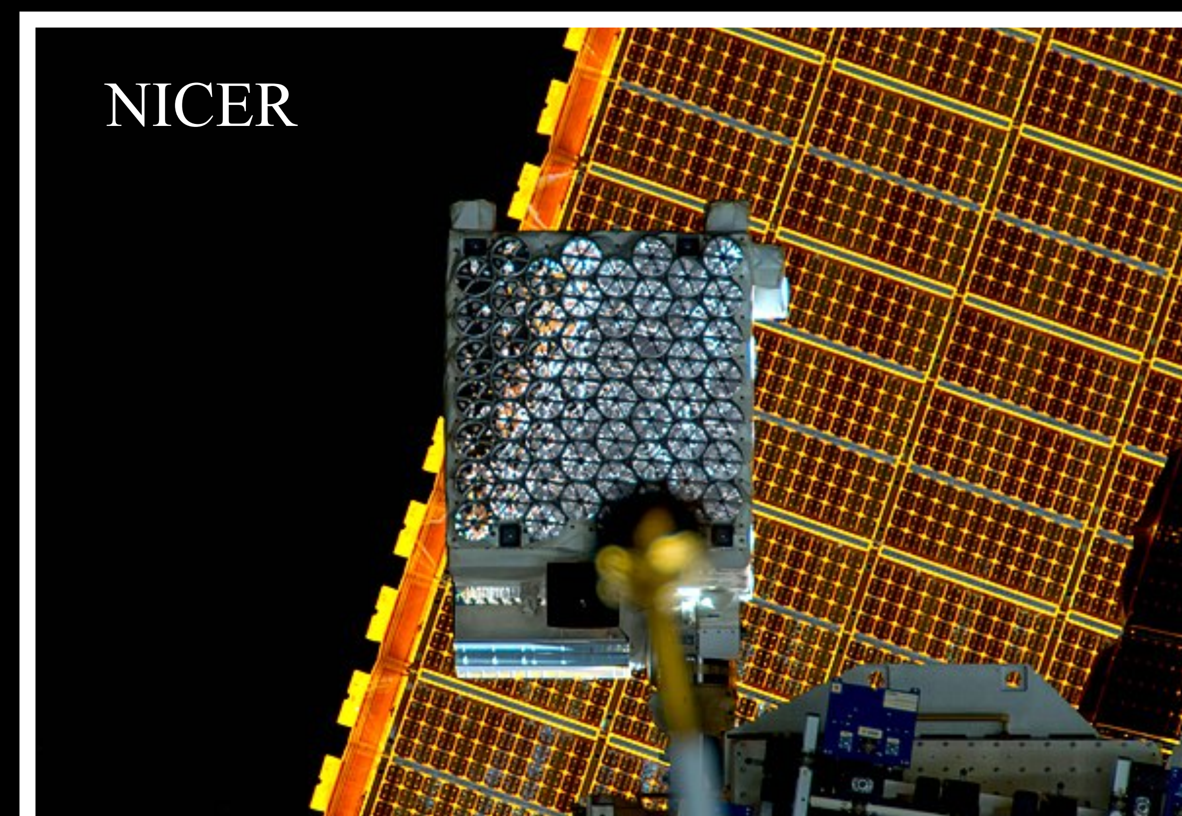
- Pulsed emission—look for effects of gravitational field (i.e. mass and radius) on time variations of flux; millisecond period X-ray pulsars (NICER)



Measure radius from X-ray timing

- Pulsed emission—look for effects of gravitational field (i.e. mass and radius) on time variations of flux; millisecond period X-ray pulsars (NICER)

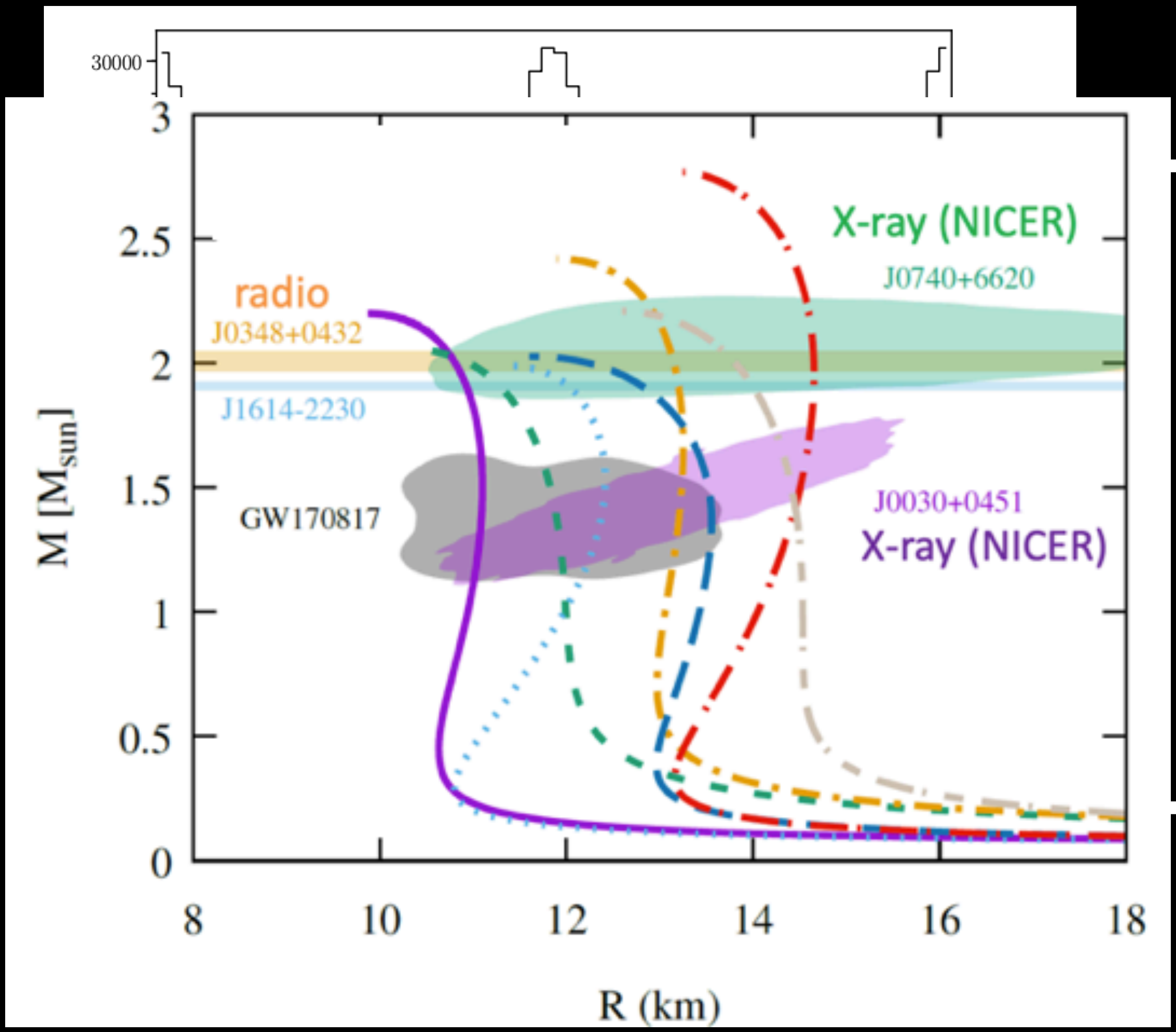


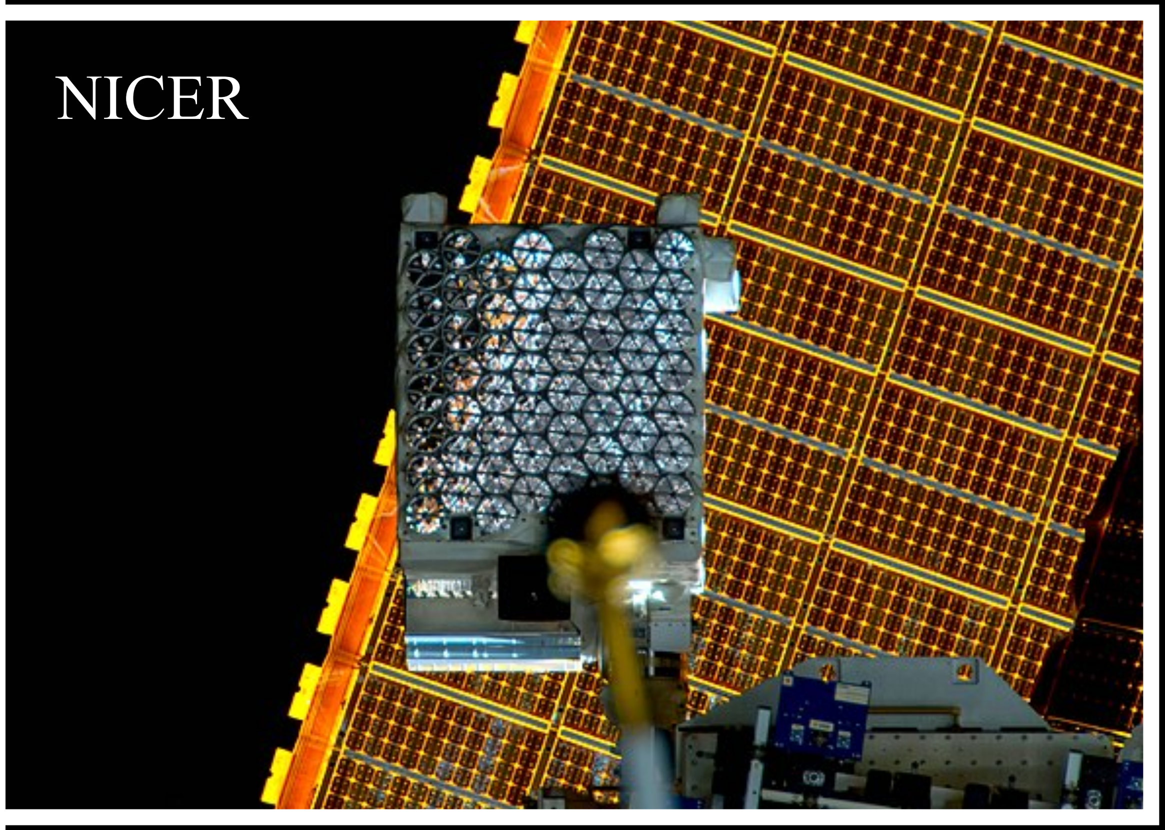


PSR J0030+0451

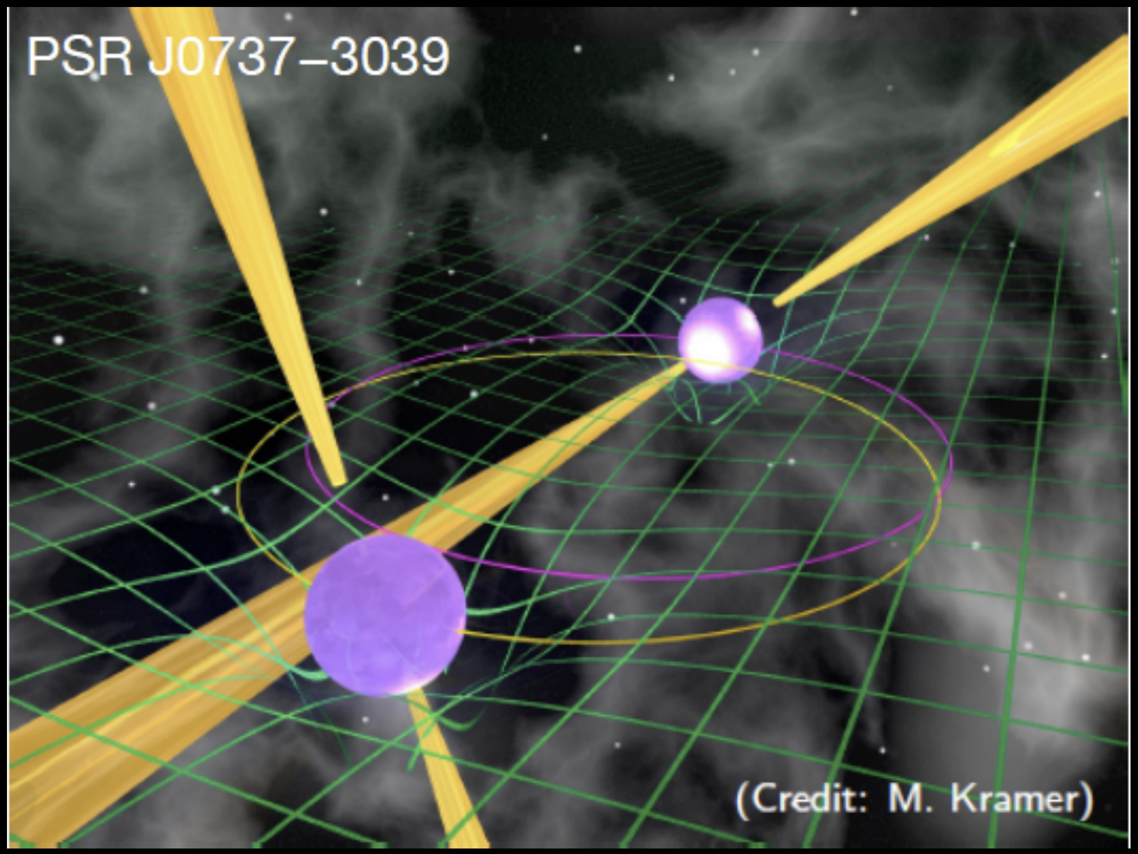
Measure radius from X-ray timing

- Pulsed emission—look for effects of gravitational field (i.e. mass and radius) on time variations of flux; millisecond period X-ray pulsars (NICER)

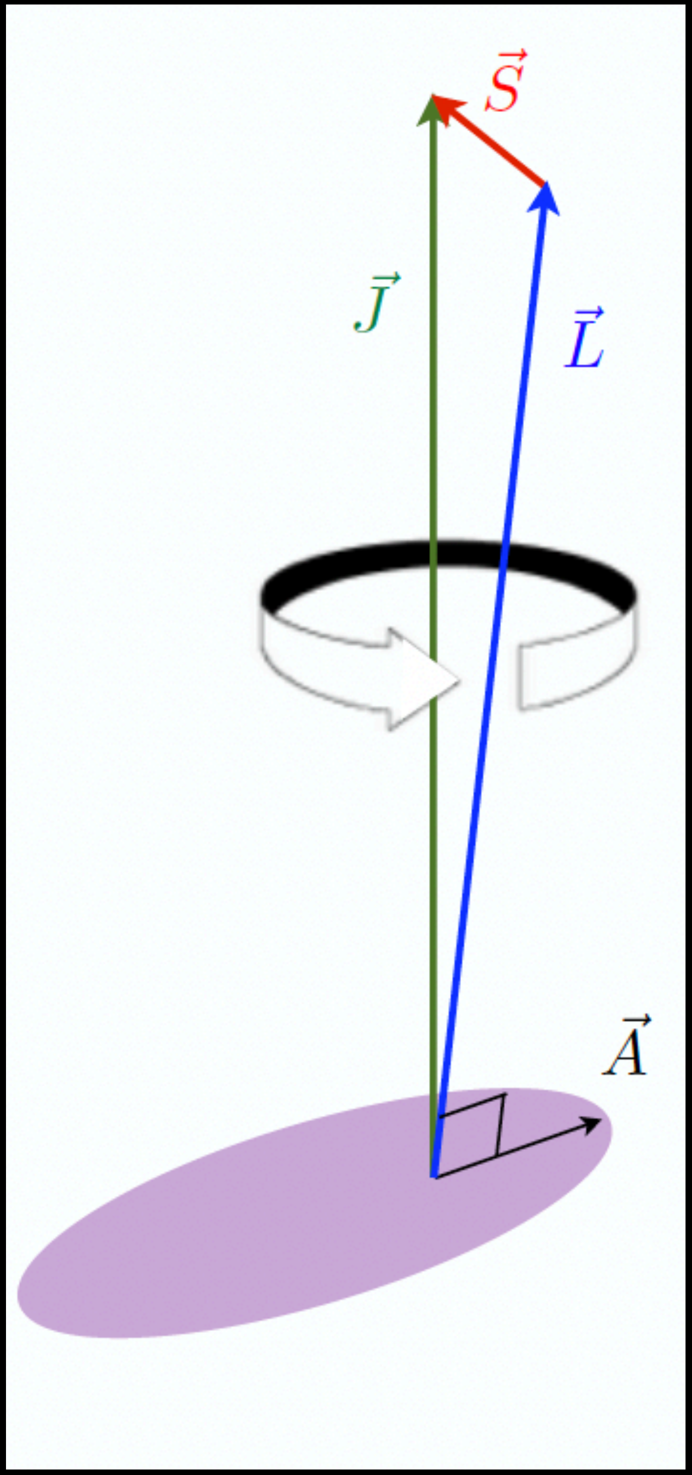




Measuring moment of inertia from frame dragging



Spin of the body causes a precession of the orbit: Lense-Thirring precession



$$\dot{\omega}^{\rm intr} = \dot{\omega}^{\rm 1PN} + \dot{\omega}^{\rm 2PN} + \dot{\omega}^{\rm LT,A}$$

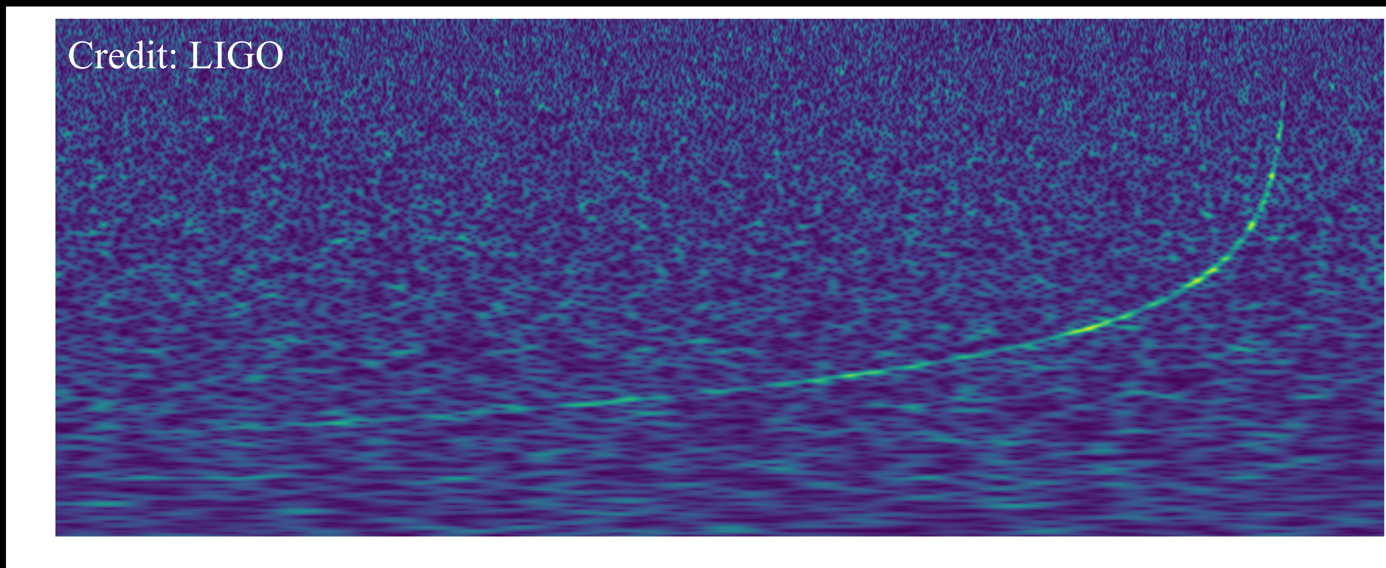
$$= \frac{3\beta_{\rm O}^2 n_{\rm b}}{1 - e_{\rm T}^2}\left[1 + f_{\rm O}\beta_{\rm O}^2 - g_{\rm S_A}^{\parallel}\beta_{\rm O}\beta_{\rm S_A}\right]$$

$$\beta_{\rm S_A} = \frac{cI_{\rm A}\Omega_{\rm A}}{Gm_{\rm A}^2}$$

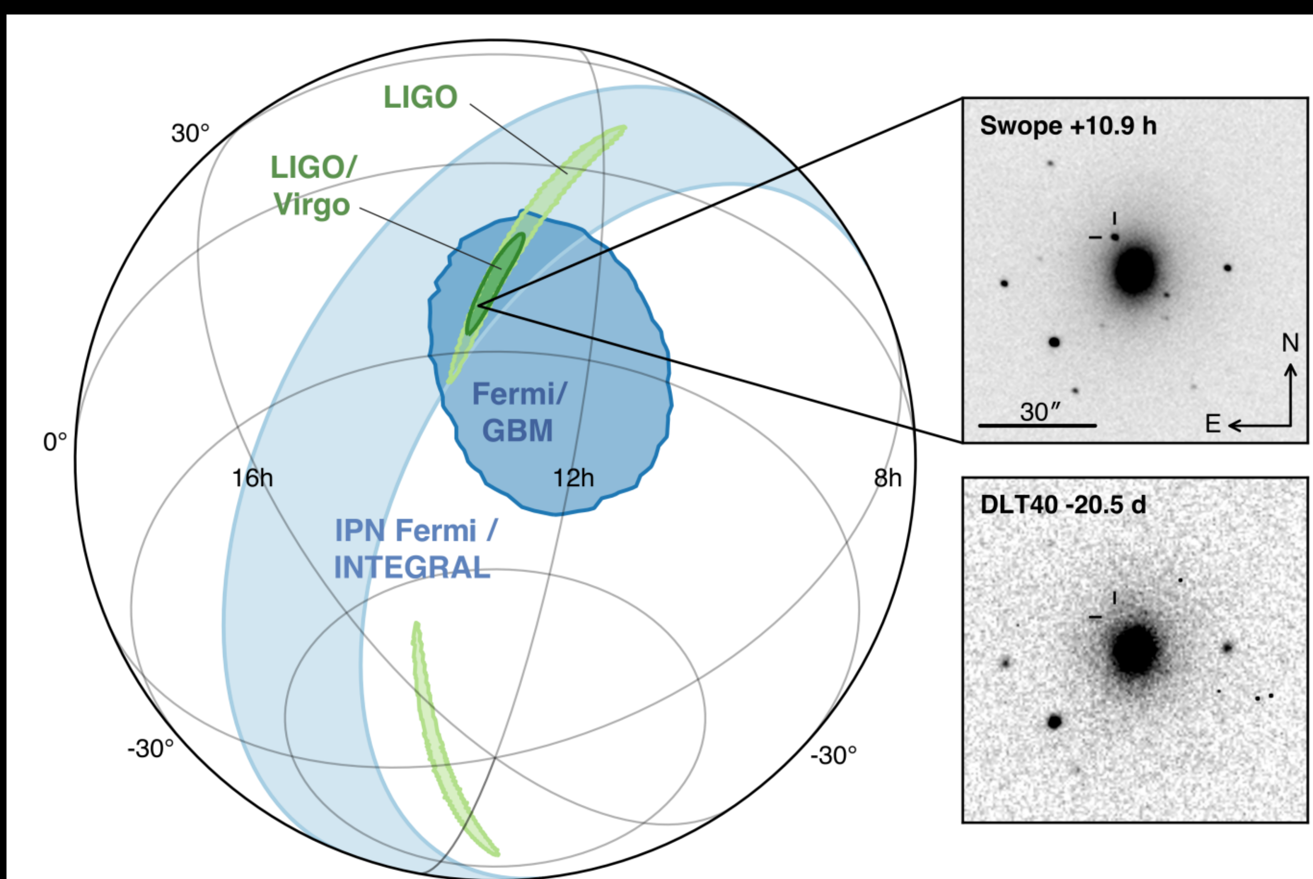
MoI dependent

$$\dot{\omega}^{\rm LT,A} \equiv n_{\rm b}k^{\rm LT,A} \simeq -3.77 \times 10^{-4} \times I_{\rm A}^{(45)}{\rm deg\,yr}^{-1}$$

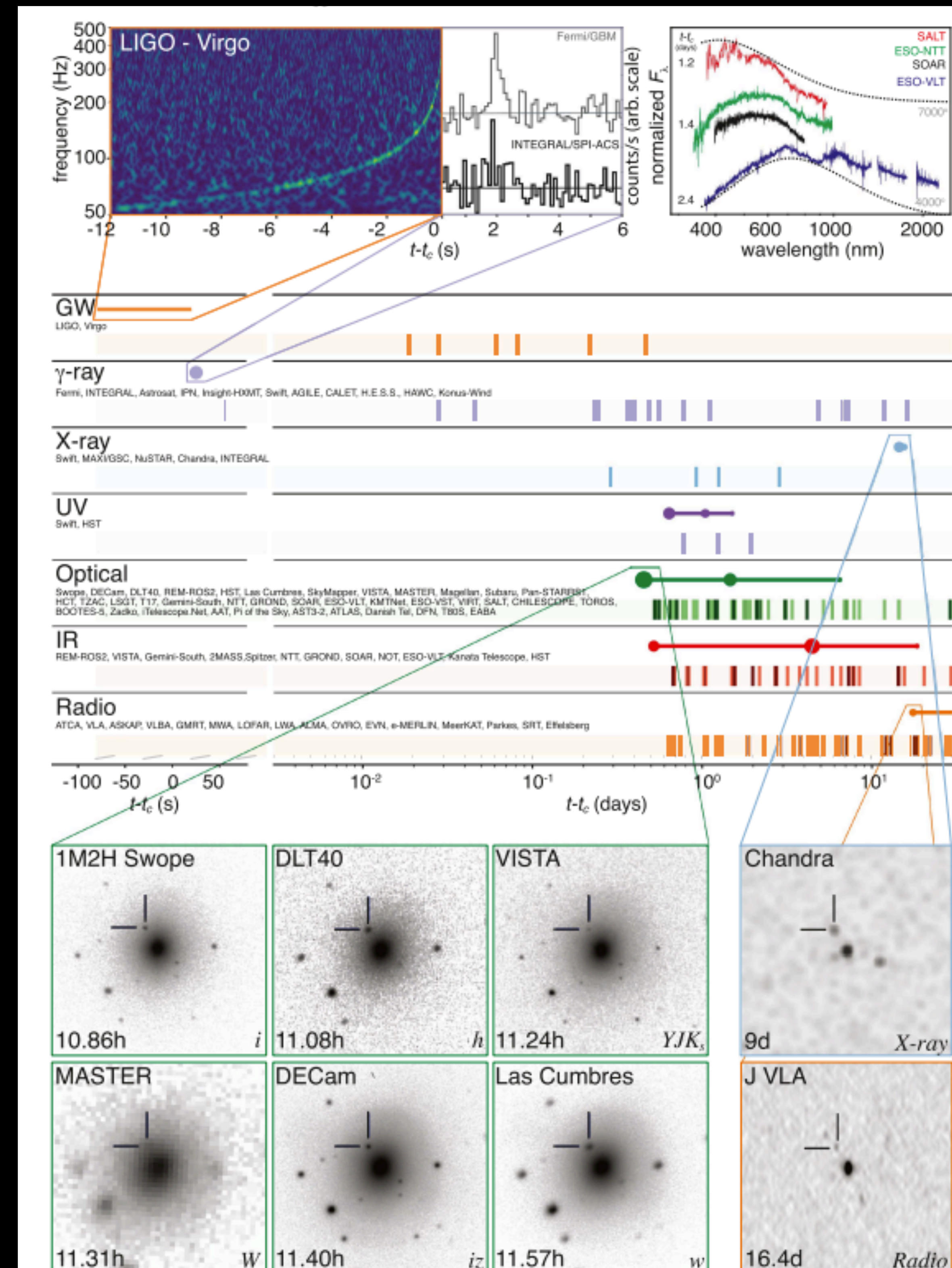
Gravitational waves and opening of multimessenger astronomy



The “chirp sound” produced as the two NSs inspiral

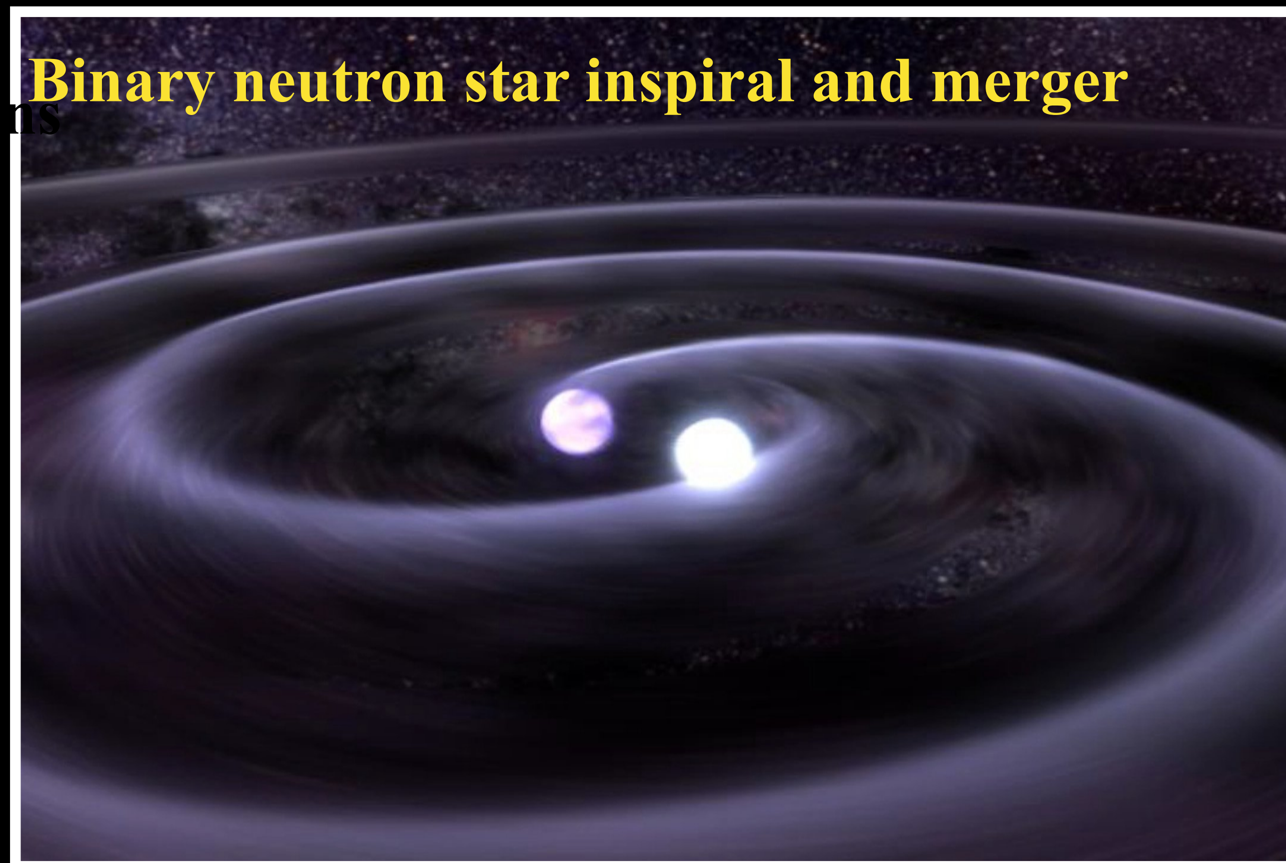


Abbott et al., PRL, 2017

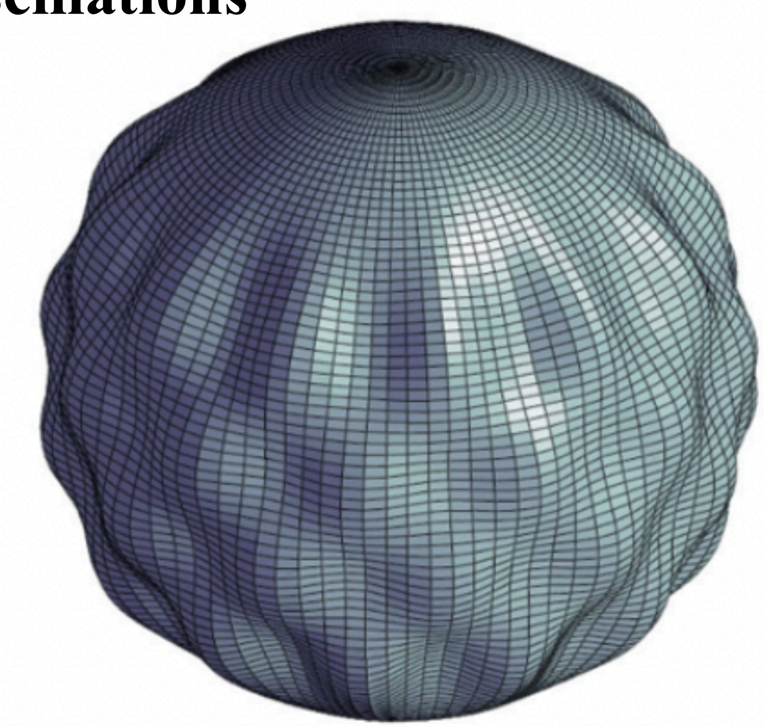


LIGO Scientific Collaboration et al., ApJL, 2017

Next lecture: Gravitational waves from NSs



Oscillations



Non-axisymmetric mass quadrupole ("mountains")

Tidal disruption in black hole-neutron star system